

ADJUSTING SERVO DRIVE COMPENSATION

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All industrial servo drives require some form of compensation often referred to as proportional, integral, and differential (PID). The process of applying this compensation is known as servo equalization or servo synthesis. In general, commercial industrial servo drives use proportional, and integral compensation (PI). It is the purpose of this discussion to analyze and describe the procedure for implementing PI servo compensation.

The block diagram of figure 1 represents dc and brushless dc motors. All commercial industrial servo drives make use of a current loop for torque regulation requirements. Figure 1 includes the current loop for the servo drive with PI compensation. Since the block diagram of figure 1 is not solvable, block diagram algebra separates the servo loops to an inner and outer servo loop of figure 2.

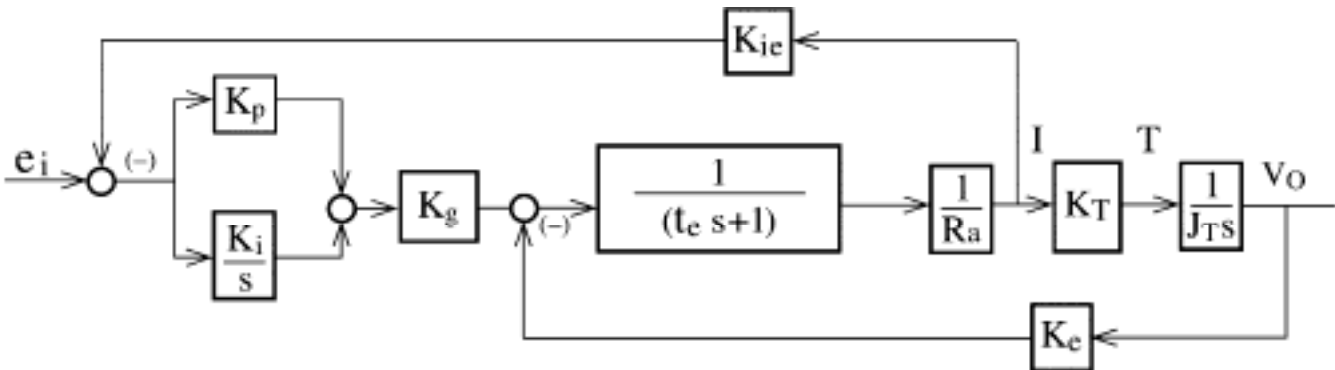


FIGURE 1 MOTOR AND CURRENT LOOP

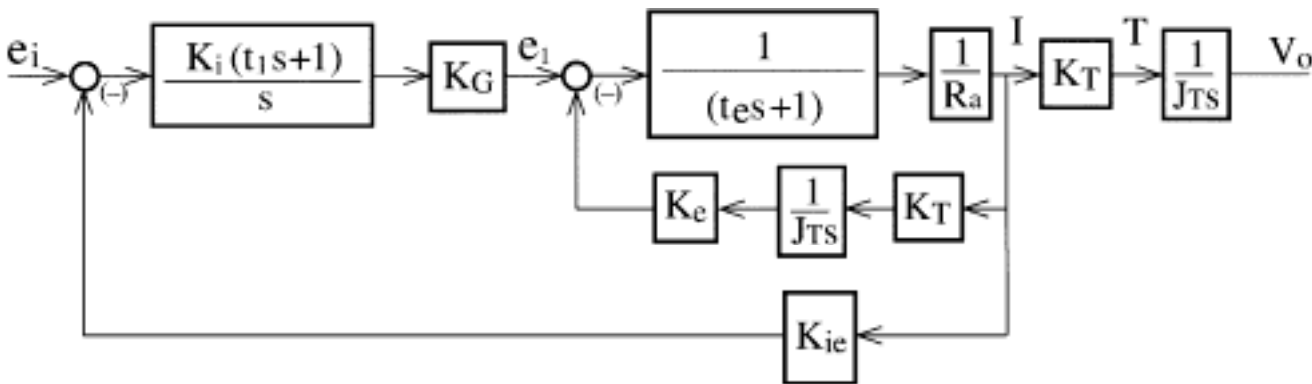


FIGURE 2 MOTOR AND CURRENT LOOP

For this discussion a worst case condition for a large industrial servo axis will be used. The following parameters are assumed from this industrial machine servo application:

Motor - Kollmorgen motor - M607B

Machine slide weight - 50,000 lbs

Ball screw: Length - 70 inches

Diameter - 3 inches

Lead - 0.375 inches/revolution

Pulley ratio - 3.333

J_T = Total inertia at the motor = 0.3511 lb-in-sec²

t_e = Electrical time constant = 0.02 second = 50 rad/sec

$t_l = t_e$

K_e = Motor voltage constant = 0.646 volt-sec/radian

K_T = Motor torque constant = 9.9 lb-in/amp

K_G = Amplifier gain = 20 volts/volt

K_{ie} = Current loop feedback constant = 3 volts/40A = 0.075 volt/amp

R_a = Motor armature circuit resistance = 0.189 ohm

K_i = Integral current gain = 735 amp/sec/radian/sec

The first step in the analysis is to solve the inner loop of figure 2. The closed loop response $I/e_1 = G/1+GH$ where:

$$G = 1/R_a (t_e S + 1) = 5.29/[t_e S + 1] \quad (5.29 = 14.4 \text{ dB})$$

$$GH = 0.646 \times 9.9/[0.189 \times 0.3511[(t_e S + 1)S]]$$

$$GH = 96/S[t_e S + 1] \quad 96 = 39 \text{ dB}$$

$$1/H = J_T S / K_e K_T = 0.3511 S / 0.646 \times 9.9$$

$$1/H = 0.054 S \quad (0.054 = -25 \text{ dB})$$

Using the rules of Bode, the resulting closed loop Bode plot for I/e_1 is shown in figure 3. Solving the closed loop mathematically :

$$\frac{I}{e_1} = \frac{G}{1 + GH} = \frac{1}{R_a(t_e S + 1) + K_e K_T / J_T S} = \frac{J_T S}{J_T R_a S(t_e S + 1) + K_e K_T}$$

$$\frac{I}{e_1} = \frac{J_T S}{J_T R_a t_e S^2 + J_T R_a S + K_e K_T} = \frac{J_T / K_e K_T S}{[(J_T R_a / K_e K_T) t_e S^2 + (J_T R_a / K_e K_T) S + 1]}$$

$$\frac{I}{e_1} = \frac{(0.3511 / 0.646 \times 9.9) S}{t_m t_e S^2 + t_m S + 1} = \frac{0.054 S}{0.01 \times 0.02 S^2 + 0.01 S + 1}$$

$$\text{where: } t_m = \frac{J_T R_a}{K_e K_T} = \frac{0.3511 \times 0.189}{0.646 \times 9.9} = 0.01 \text{ sec, } \omega_m = 1/t_m = 100 \text{ rad/sec}$$

$$t_e = 0.02 \text{ sec} \quad \omega_e = 1/t_e = 50 \text{ rad/sec}$$

For a general quadratic-

$$\frac{S^2}{\omega_r^2} + \frac{2 \text{ delta}}{\omega_r} S + 1$$

$$\omega_r = [\omega_m \omega_e]^{1/2} = [100 \times 50]^{1/2} = 70 \text{ rad/sec}$$

$$\frac{I}{e_1} = \frac{0.054 S}{S^2 / 70^2 + (2 \text{ delta} / 70) S + 1}$$

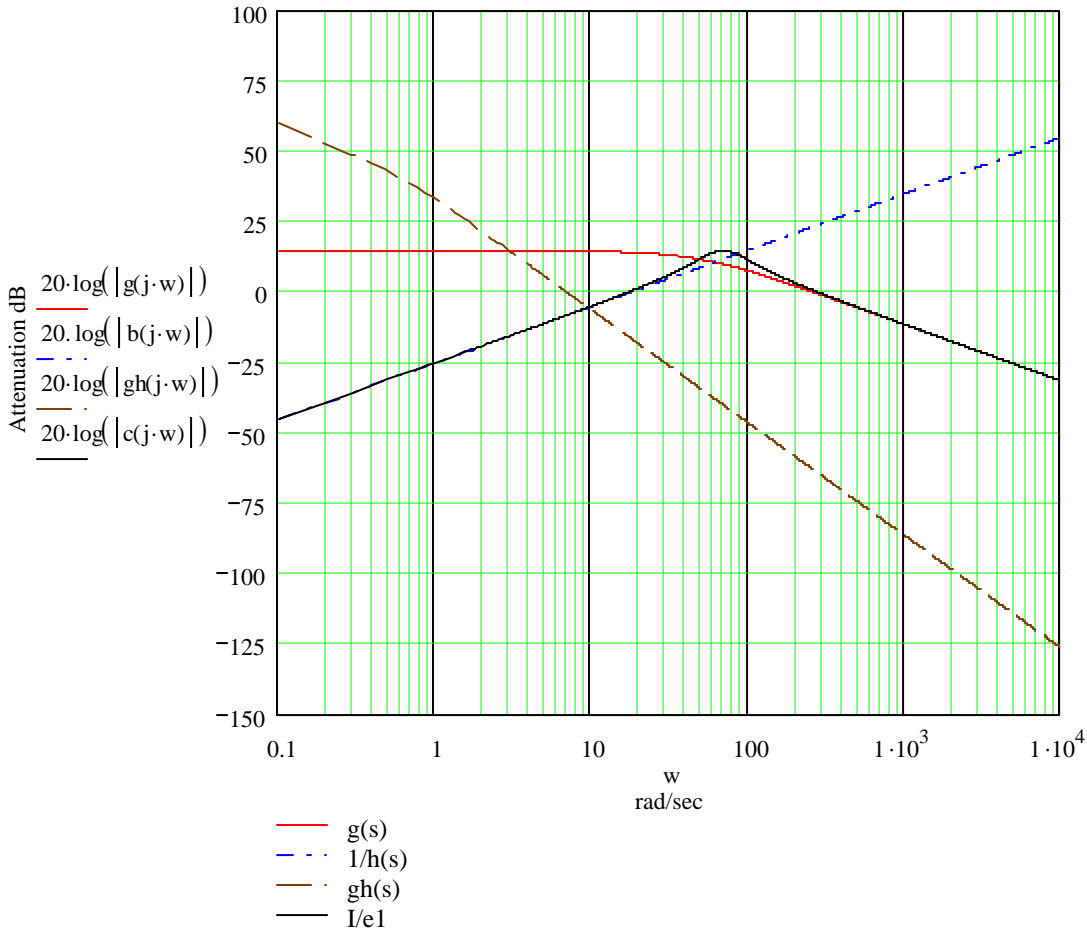


Fig 3 Current inner loop

Having solved the inner servo loop it is now required to solve the outer current loop. The inner servo loop is shown as part of the current loop in figure 4.

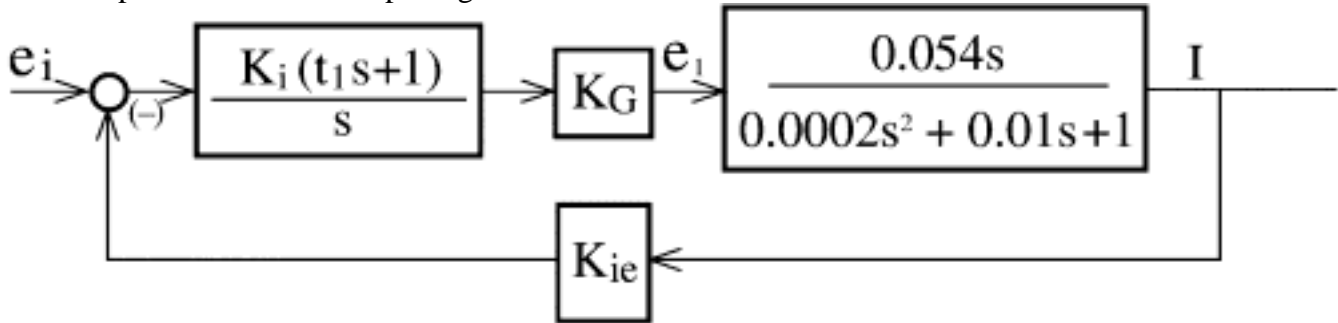


FIGURE 4 CURRENT LOOP

In solving the current loop, the forward loop, open loop, and feedback loop must be identified as follows:

The forward servo loop-

$$G = \frac{K_i K_G \times 0.054 (0.02S+1)}{0.0002 S^2 + 0.01 S + 1} = \frac{735 \times 20 \times 0.054 (0.02S+1)}{0.0002S^2 + 0.01 S + 1}$$

$$G = \frac{794(0.02 S+1)}{0.0002S^2 + 0.01S + 1} = \frac{15.88S + 794}{0.0002S^2 + 0.01S + 1}$$

Where: $K_G = 20$ volt/volt

$K_{ie} = 3/40 = 0.075$ volt/amp

$K_i K_G \times 0.054 = 794$ (58 dB)

$K_i = 794/(20 \times 0.054) = 735$

$$G = \frac{79,400S + 3,970,000}{S^2 + 50 S + 5000}$$

The open loop-

$$GH = 0.075 \times \frac{79,400S + 3,970,000}{S^2 + 50S + 5000}$$

$$GH = \frac{5955S + 297,750}{S^2 + 50S + 5000}$$

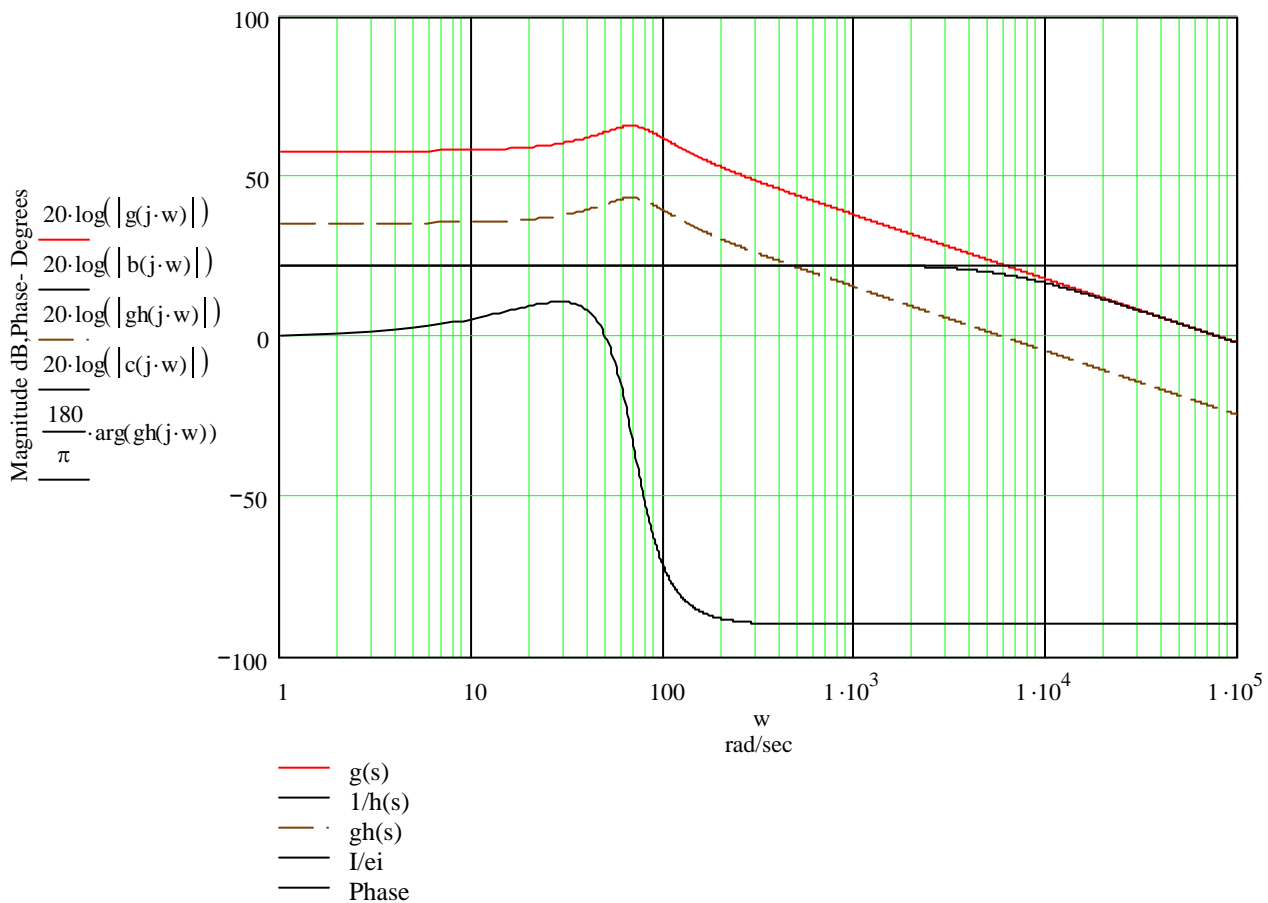


Fig 5 Current loop response

The feedback current scaling is-

$$H = 3 \text{ volts/40 amps} = 0.075 \text{ volts/amp} \quad 1/H = 13.33 = 22.4 \text{ dB}$$

The Bode plot frequency response is shown in figure 5. The current loop bandwidth is 6000 radians/second or about 1000 Hz, which is realistic for commercial industrial servo drives.

The current loop as shown in figure 5 can now be included in the motor servo loop with reference to figure 2 and reduces to the motor servo loop block diagram of figure 6.

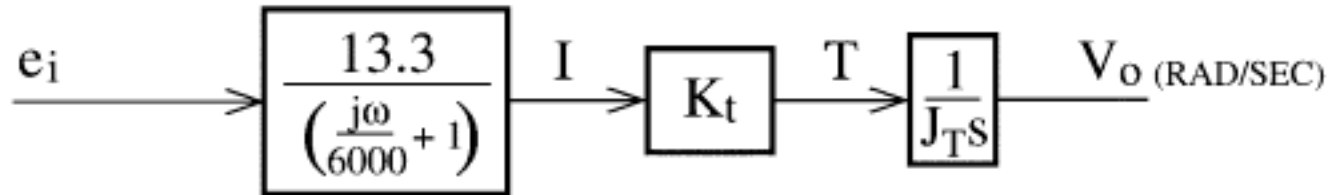


FIGURE 6

The completed motor servo loop has a forward loop only (as shown in figure 6) where:

$$J_T = \text{Total inertia at the motor} = 0.3511 \text{ lb-in-sec}^2$$

$$K_T = \text{Motor torque constant} = 9.9 \text{ lb-in/amp}$$

$$G = \frac{13.3 \times 9.9}{.3511S ((jw/6000) + 1)} = \frac{375}{S(0.000166S + 1)} \quad (51.5 \text{ dB})$$

$$G = \frac{375}{0.000166S^2 + S + 0} = \frac{2,250,090}{S^2 + 6000S + 0}$$

$$\frac{v_o}{e_i} = \frac{K_T}{J_T S} \times \frac{|I|}{|V_i|} = \frac{9.9}{0.3511S} \times \frac{13.1(0.02S + 1)}{0.00000331S^2 + 0.02S + 1}$$

$$\frac{v_o}{e_i} = \frac{375(0.02S + 1)}{S \cdot 0.00000331(S + 50)(S + 5991)}$$

$$\frac{v_o}{e_i} = \frac{375(0.02S + 1)}{S \cdot 0.00000331 \times 50 \times 5991 ((S/50) + 1)((S/5991) + 1)}$$

$$\frac{v_o}{e_i} = \frac{375(0.02S + 1)}{S(0.02S + 1)(0.000166S + 1)}$$

$$\frac{v_o}{e_i} = \frac{375}{S((jw/5991) + 1)}$$

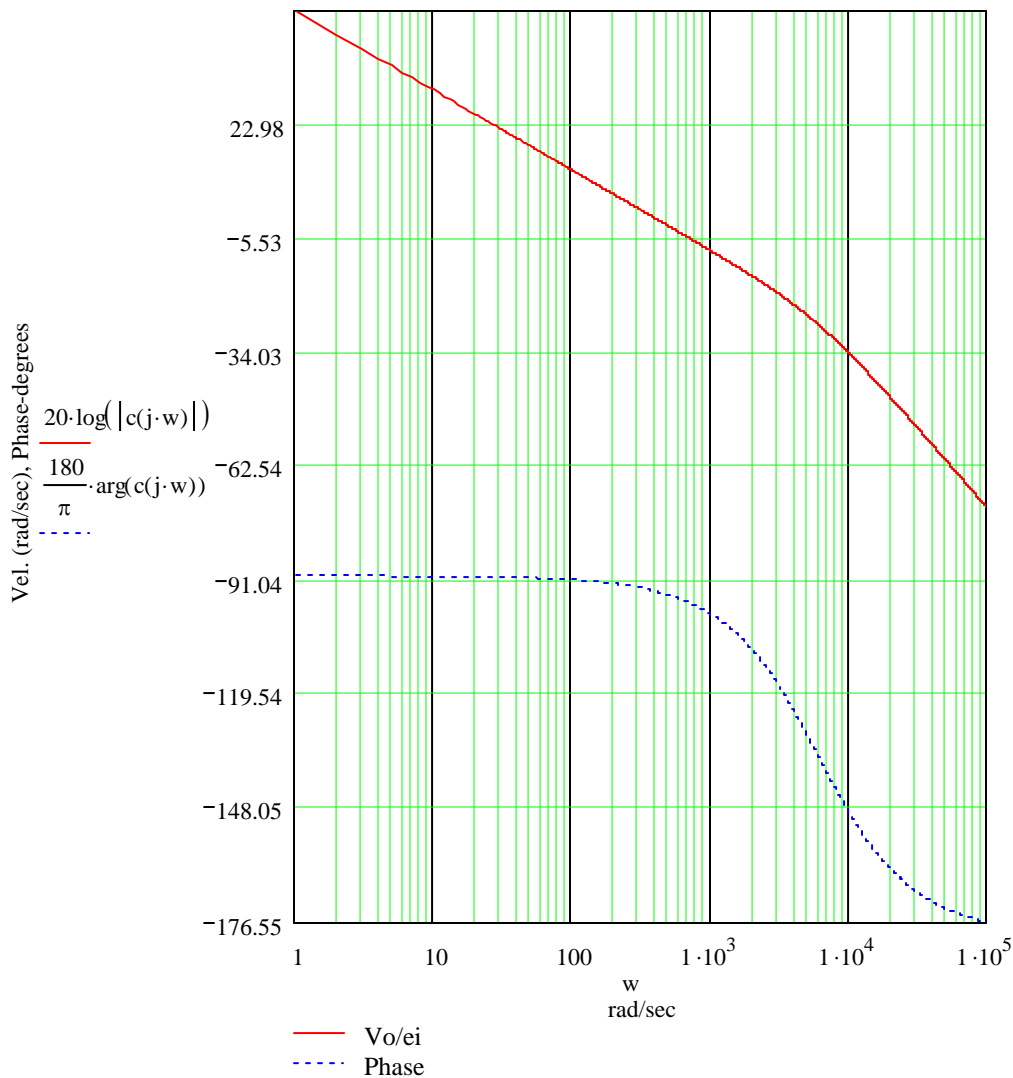


Fig 7 Mot.&I loop freq. response

The Bode frequency response for the motor and current loop is shown in figure 7. The motor and current closed loop frequency response, indicate that the response is an integration which includes the 6000 rad/sec bandwidth of the current loop. This is a realistic bandwidth for commercial industrial servo drives.

Usually this response is enclosed in a velocity loop and further enclosed in a position loop. However, there are some applications where the motor and current loop are enclosed in a position loop. Such an arrangement is shown in figure 7a.

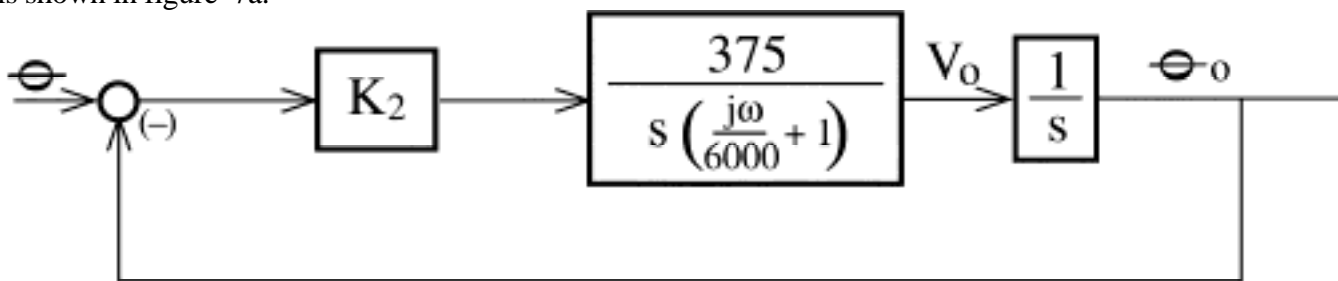


FIGURE 7a POSITION LOOP

The forward loop transfer function is

$$G = \frac{K_v}{S^2((j\omega/6000) + 1)}$$

Where: $K_v = K_2 \times 375$

For most large industrial machine servo drives a position loop $k_v = 1$ ipm/mil or 16.6 rad/sec can be assumed. The frequency response for the position loop is shown in figure 7b. This response is obviously unstable with a minus 2 slope at the zero gain point. It is also obvious that there are two integrators in series, resulting in an oscillator. If a velocity loop is not used around the motor and current loop, some form of differential function is required to obtain stability. By adding a differential term at about 10g rad/sec in figure 7b, the response can be modified to that of a type 2 control which could have performance advantages. The absence of the minor velocity servo loop bandwidth could make it possible to increase the position loop velocity constant (position loop gain) for an increase in the position loop response.

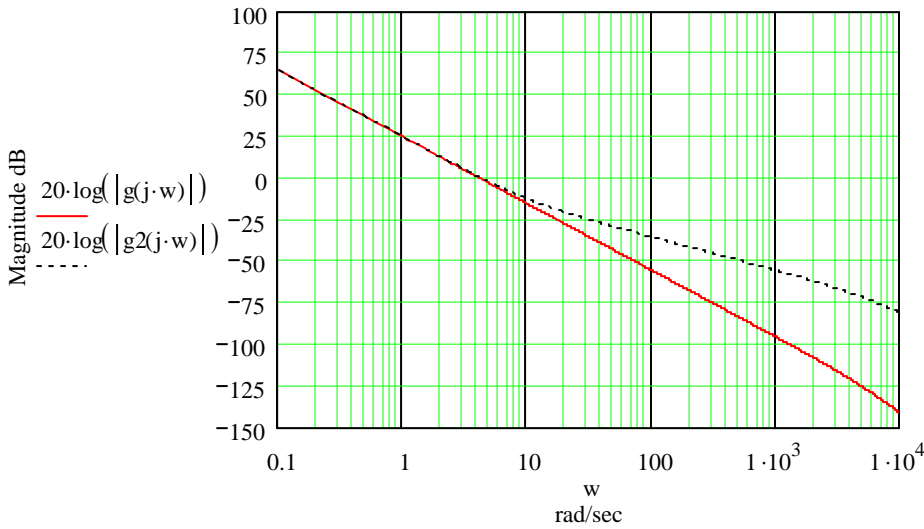


Fig. 7b Position loop response

For the purposes of this discussion it will be assumed that the motor and current loop are enclosed in a velocity servo loop. Such an arrangement is shown in figure 8.

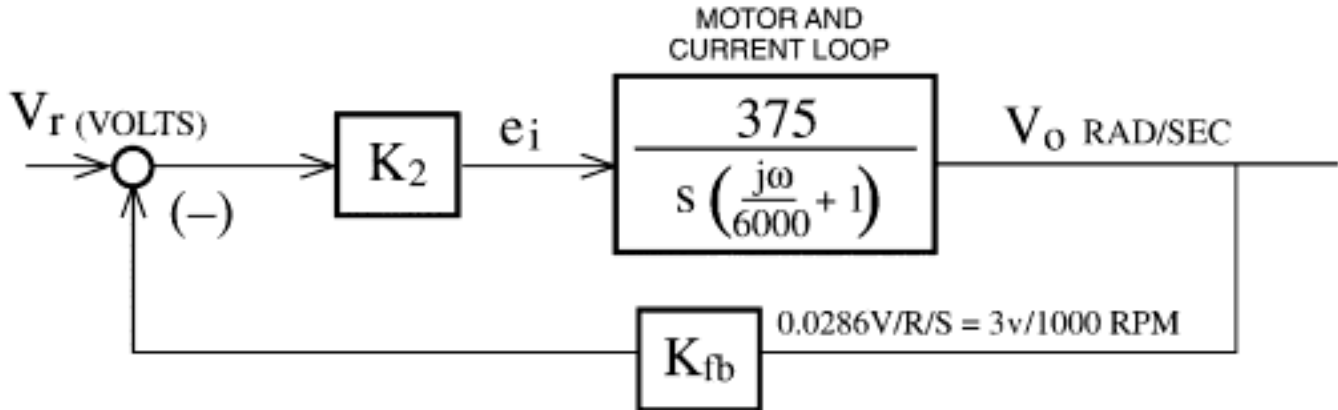


FIGURE 8 VELOCITY LOOP

The servo compensation and amplifier gain are part of the block identified as K_2 . Most industrial servo drives use proportional plus integral (PI) compensation. The amplifier and PI compensation can be represented as in figure 9¹.

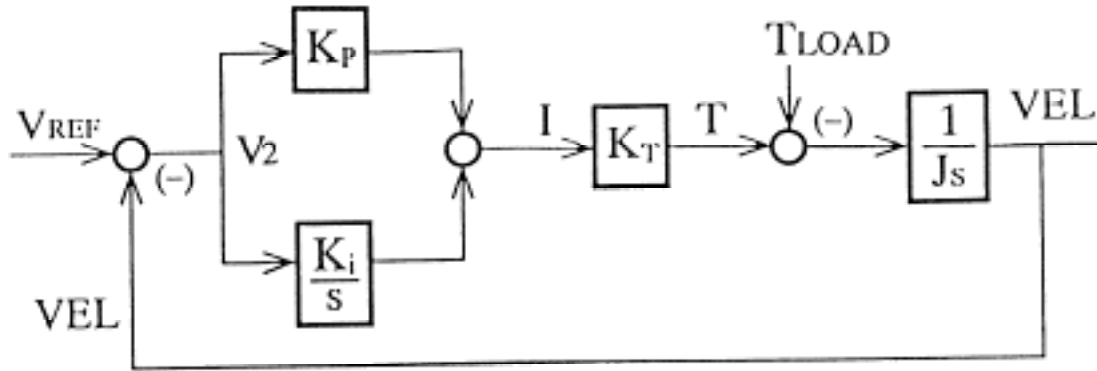


Figure 9

$$\frac{I}{V_2} = \left[K_p + \frac{K_i}{s} \right] = \frac{K_p s + K_i}{s} = \frac{K_i \left[\frac{K_p}{K_i} s + 1 \right]}{s} = \frac{K_2 [t_2 s + 1]}{s}$$

$$t_2 = \frac{K_p}{K_i} \quad \omega_2 = \frac{K_i}{K_p} \quad (\text{Corner frequency})$$

The adjustment of the PI compensation is suggested as-

1. For the uncompensated servo Bode plot, set the amplifier gain to a value just below the level of instability.
2. Note the forward loop frequency (ω_g) at -135 degree phase shift (45 degrees phase margin) of figure 12.
3. From the Bode plot for PI compensation of figure 10, the corner frequency $\omega_2 = K_i/K_p$ should be approximately $\omega_g/10$ or smaller as a figure of merit¹. The reason for this is that the attenuation characteristic of the PI controller has a phase lag that is detrimental to the servo phase margin. Thus the corner frequency of the PI compensation should be lowered about one decade or more from the -135 degree phase shift point (ω_g) of the open loop Bode plot for the servo drive being compensated. For the servo drive being considered, ω_g occurs at 6000 rad/sec.

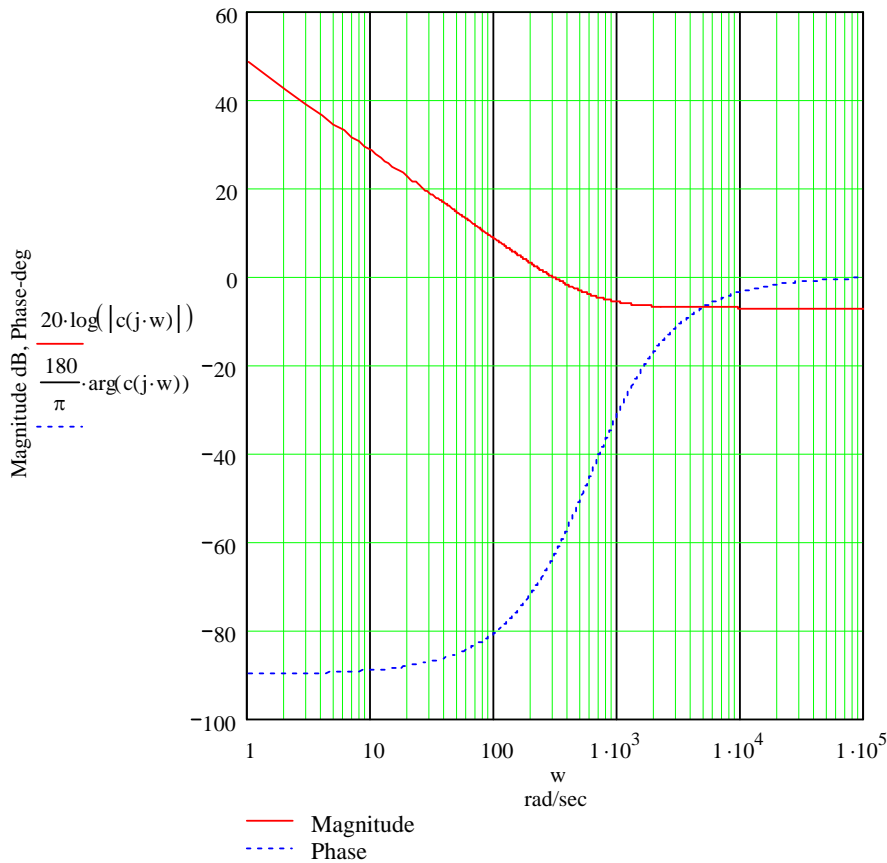


Fig 10 PI Compensation

Applying the PI compensation of figure 9, to the velocity servo drive is shown if figure 11.

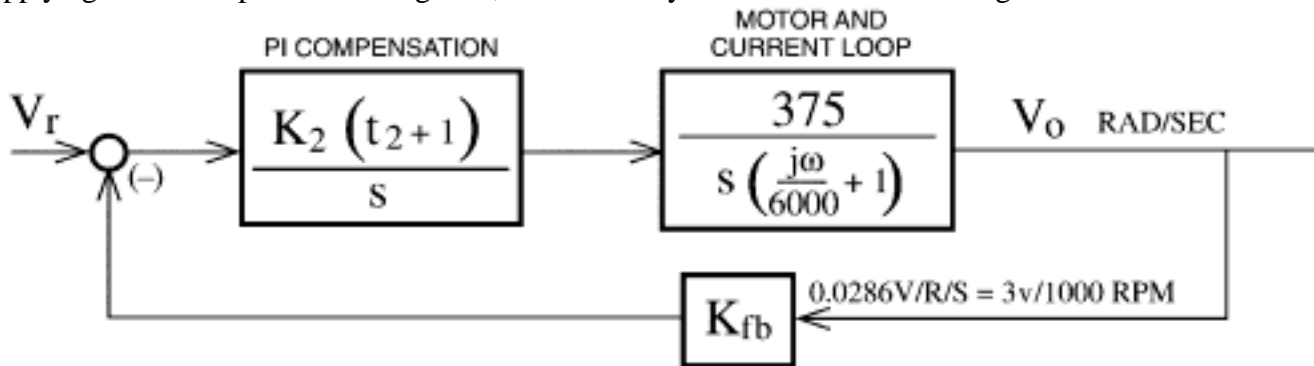


FIGURE 11 VELOCITY LOOP WITH PI COMPENSATION

In general the accepted rule for setting the servo compensation begins by removing the integral and/or differential compensation. The proportional gain is then adjusted to a level where the velocity servo response is just stable. The proportional gain is then reduced slightly further for a margin of safety.

For a gain $K_2=1000$ of the uncompensated servo, the Bode plot is shown in figure 12. It should be noted that the motor and current loop have a bandwidth of 6000 rad/sec as shown in figure 7. This is a normal response for industrial servo drive current loops. The transient response for this servo is shown in figure

13 as a damped oscillatory response. If the gain K_2 is reduced to a value of 266 for a forward loop gain of 100,000, the Bode plot is shown in figure 14 with a stable transient response shown in figure 15.

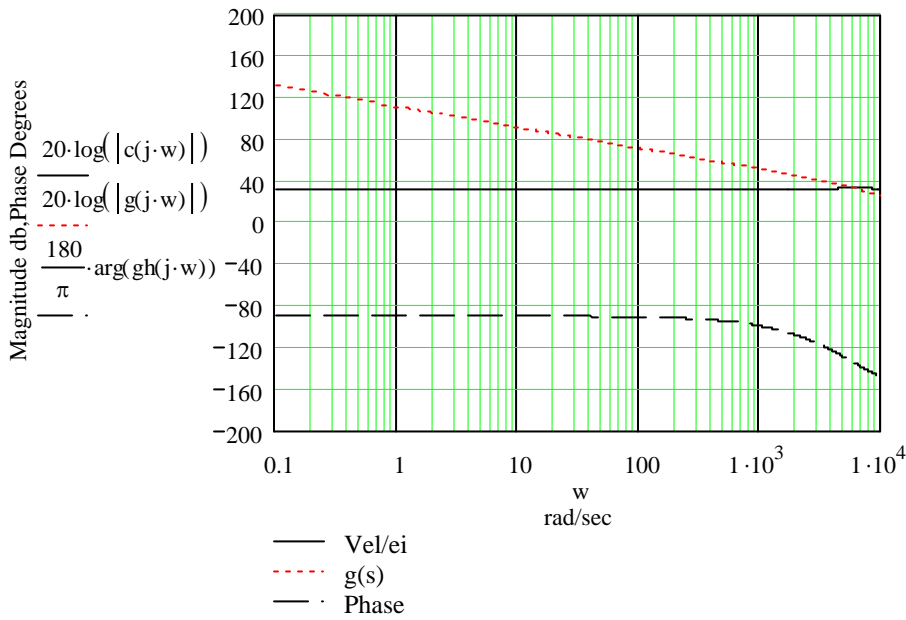


Fig 12 Velocity loop response

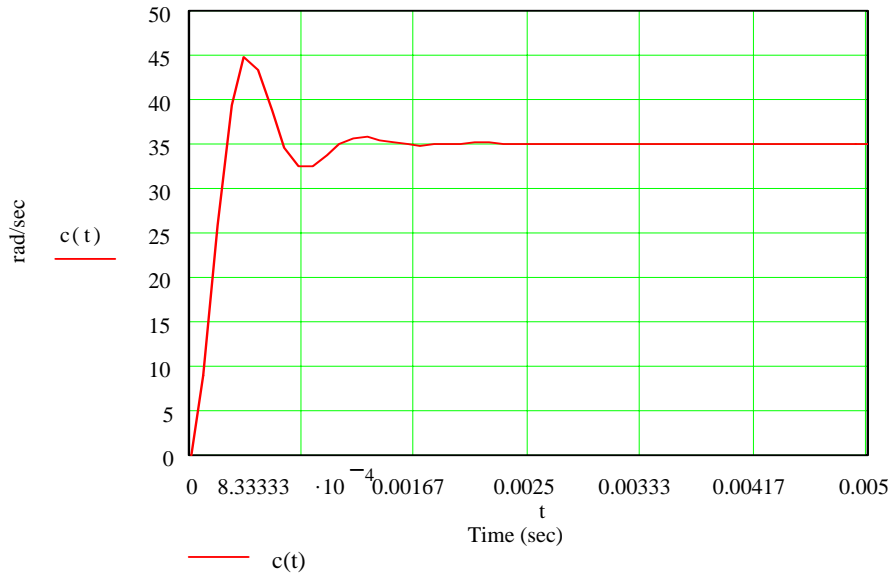


Fig 13 Transient response

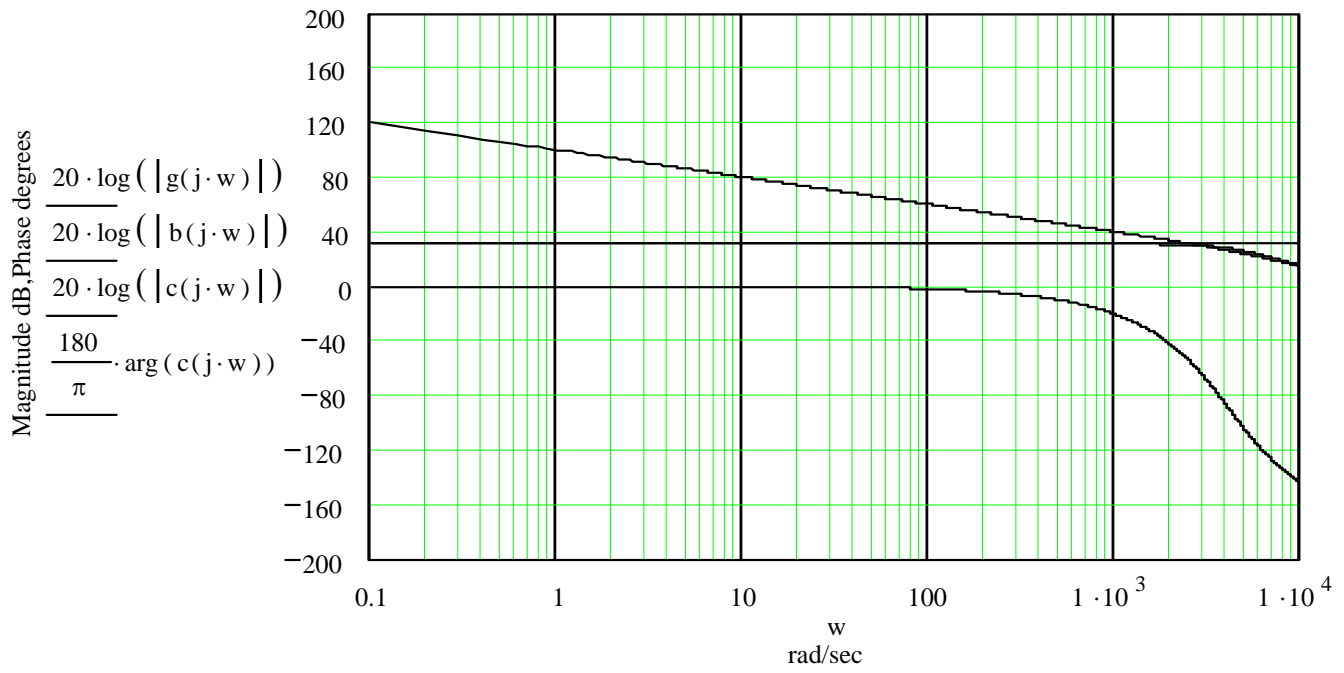


Fig 14 Velocity loop response

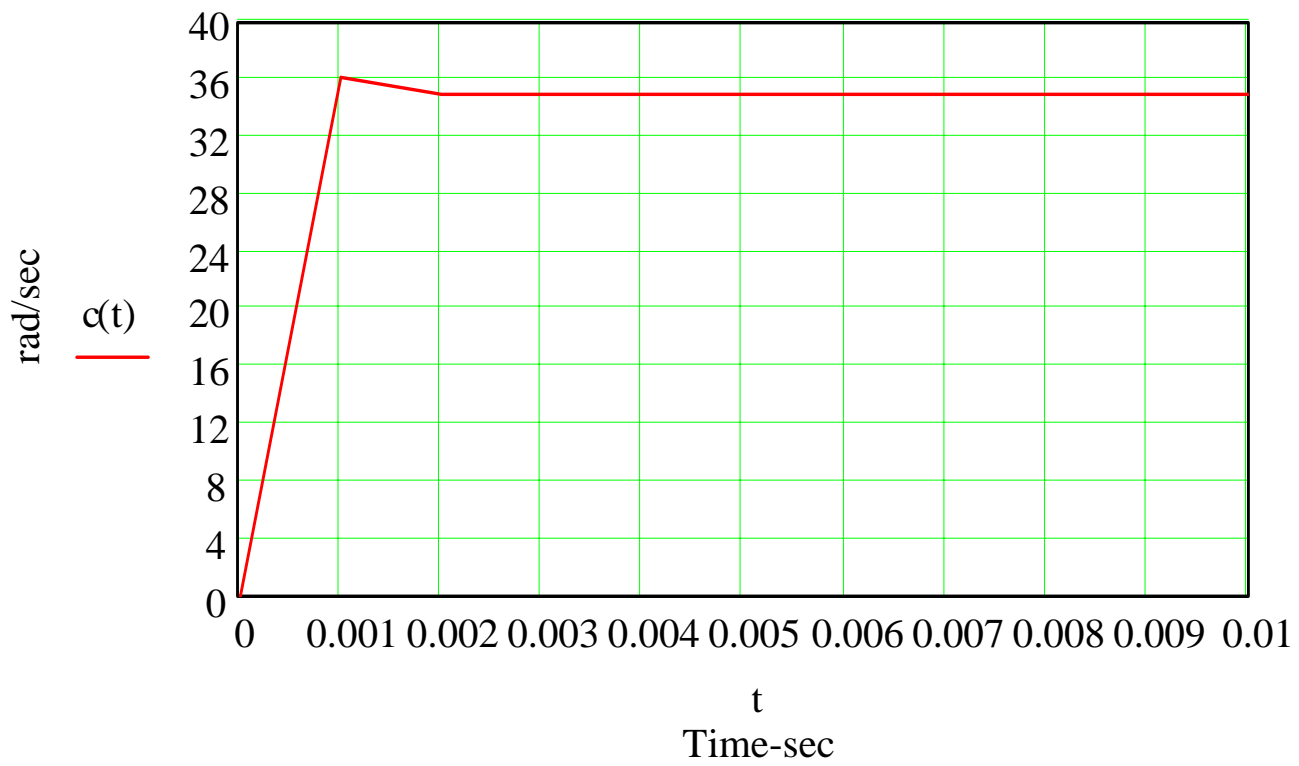


Fig 15 Transient response

At this point the PI compensation is added as shown in figure 11. The index of performance for the PI compensation is that the corner frequency $w_2 = K_i/K_p$, should be a decade or more lower than the -135 degree phase shift (45 degree phase margin) frequency (w_g) of the forward loop Bode plot (figure 14) for the industrial servo drive being considered¹.

With reference to figure 14 of the stable uncompensated servo, the -135 degree phase shift (45 degree phase margin) occurs at 6000 rad/sec frequency which is also the bandwidth of the motor/current loop. Using the index of performance of setting the PI compensation corner frequency at one decade or more lower in frequency than the -135 degree phase shift frequency point, the corner frequency should be set at 600 rad/sec or lower. With the corner frequency of the PI compensation set at 600 rad/sec (0.001666 sec) the compensated servo is shown in the Bode plot of figure 16. The transient response is shown in figure 17 as a highly oscillatory velocity servo drive. Obviously this servo drive needs to have the PI compensation corner frequency much lower than one decade (600 rad/sec) index of performance. For a two decade, 60 rad/sec (0.0166 sec) lower setting for the PI corner frequency the Bode response is shown in figure 18 with a transient response shown in figure 19 having a single overshoot in the output of the velocity servo drive.

By lowering the PI compensation corner frequency ($w_2 = \frac{K_i}{K_p}$) to 20 rad/sec (0.05 sec), a stable velocity servo drive results. The forward loop and open loop are defined as follows:

$$H = 0.0286 \text{ v/rad/sec} \quad 1/H = 34.9 \text{ (30.8 dB)}$$

$$\text{Gain @ } w=1 \text{ rad/sec} = 100 \text{ dB} = 100,000$$

$$K_2 = 100,000/375 = 266$$

$$G = \frac{K_2 \times 375 ((jw/20)+1)}{S^2 ((jw/6000)+1)} = \frac{100,000 ((jw/20)+1)}{S^2 ((jw/6000)+1)} \quad (100 \text{ dB})$$

$$G = \frac{100,000 (0.05S+1)}{S^2 (0.000166S +1)} = \frac{5000S + 100,000}{S^2 (0.000166S +1)}$$

$$G = \frac{30,120,481S + 602,409,638}{S^3 + 6024S^2 + 0S + 0}$$

$$GH = 0.0286 \times G = \frac{2860 ((jw/20)+1)}{S^2 ((jw/6000)+1)} \quad (69 \text{ dB})$$

The Bode plot for the velocity loop with PI compensation is shown in figure 20, having a typical industrial velocity servo bandwidth of 30 Hz (188 rad/sec). The transient response is stable with a slight overshoot in velocity as shown in figure 21.

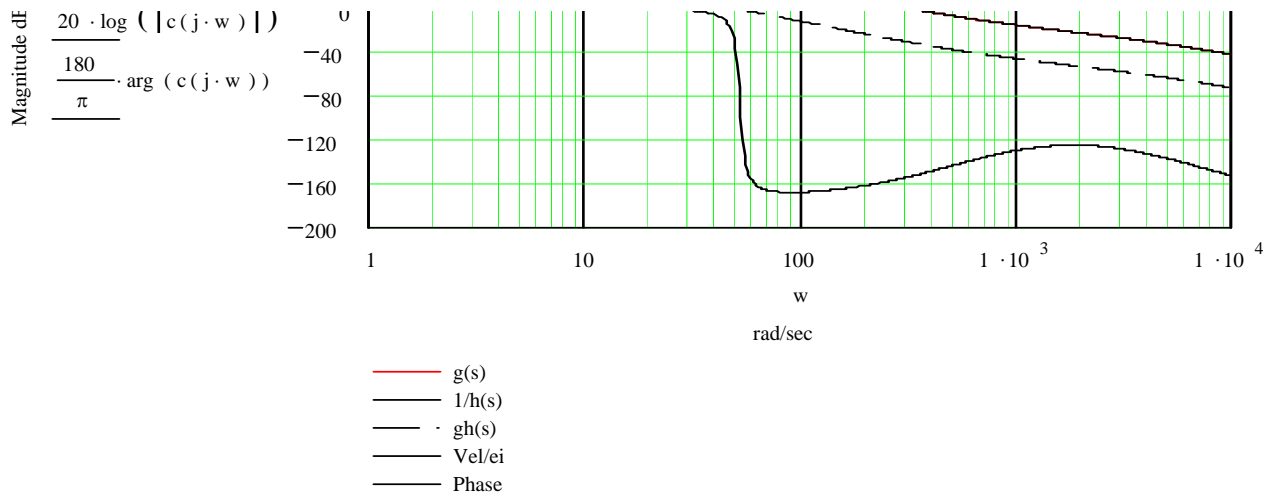


Fig 16 Velocity loop response

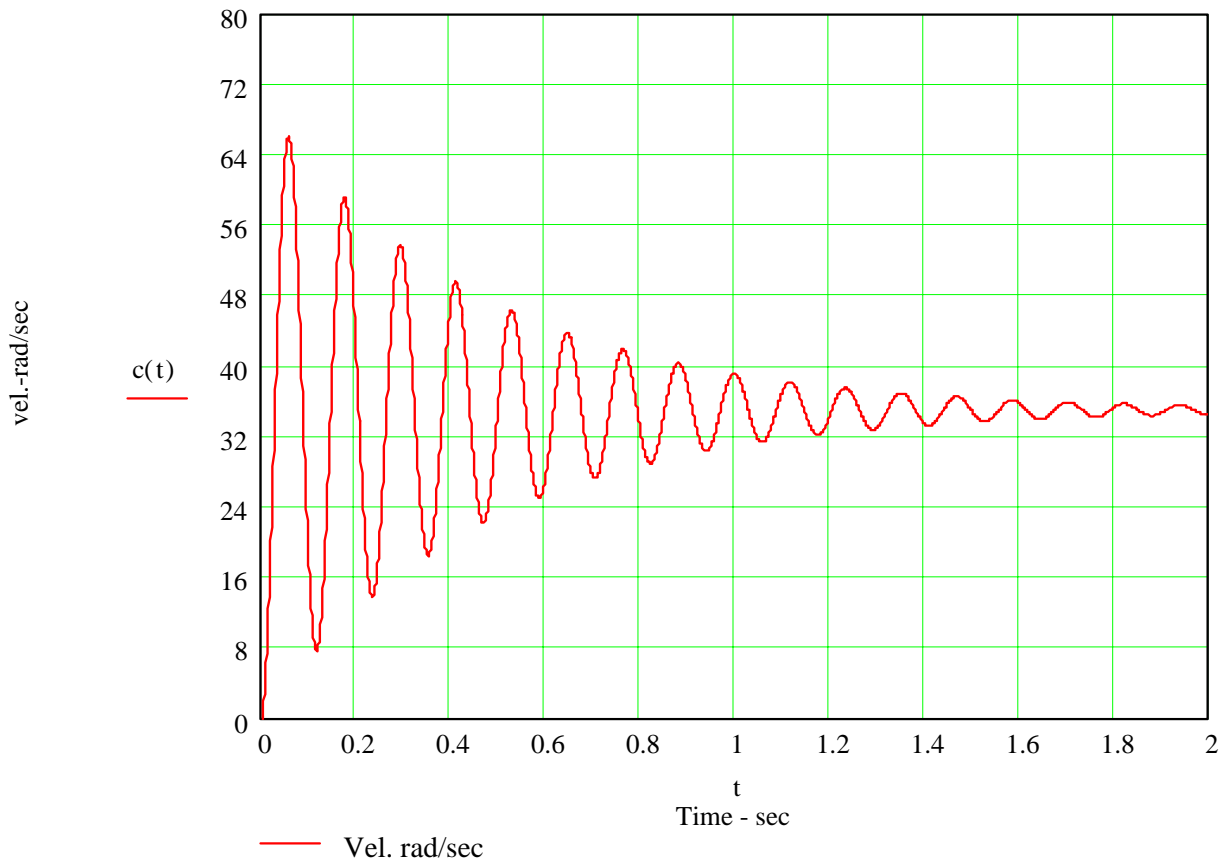


Fig 17 Vel. servo trans. resp.

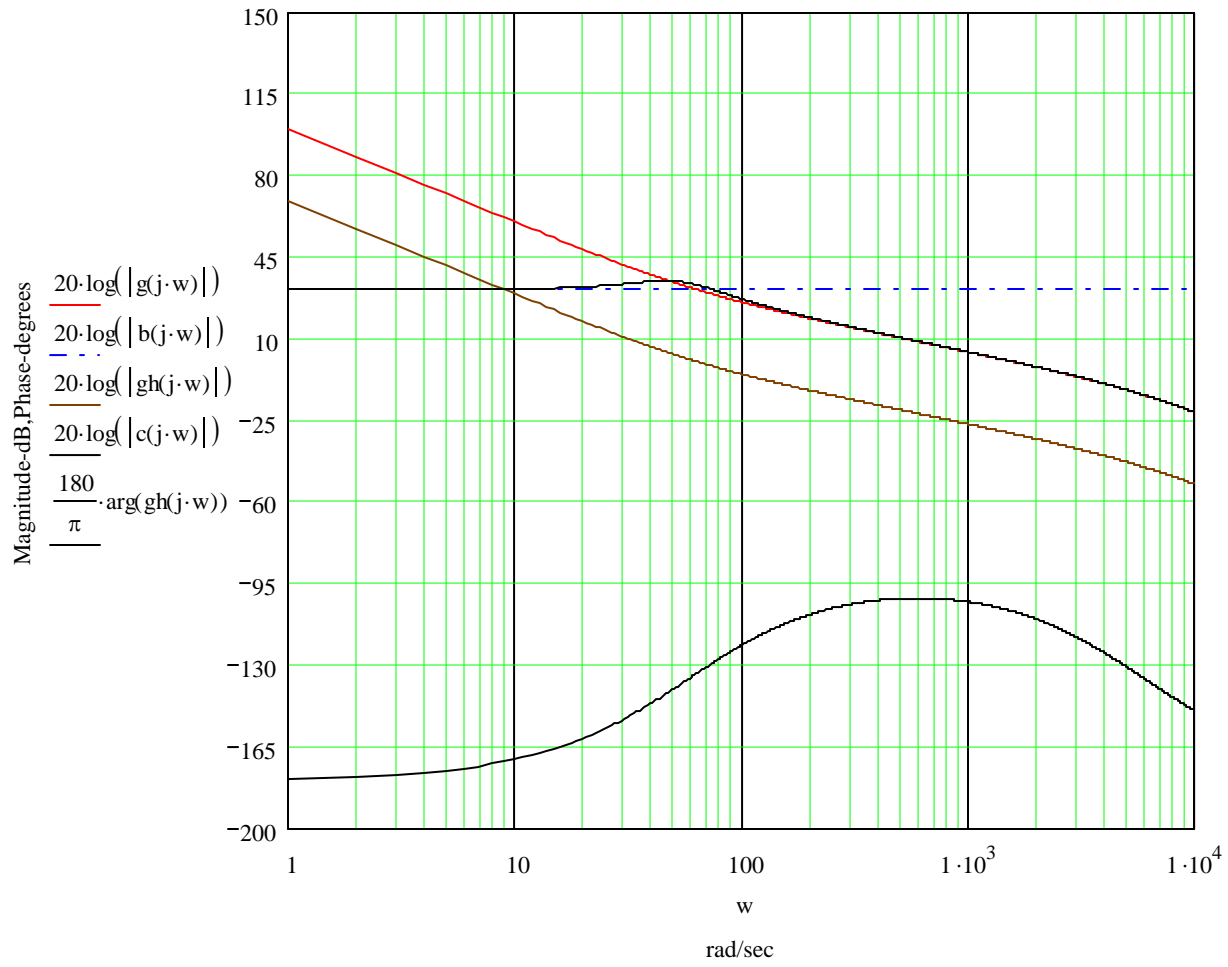


Fig 18 Vel. loop response

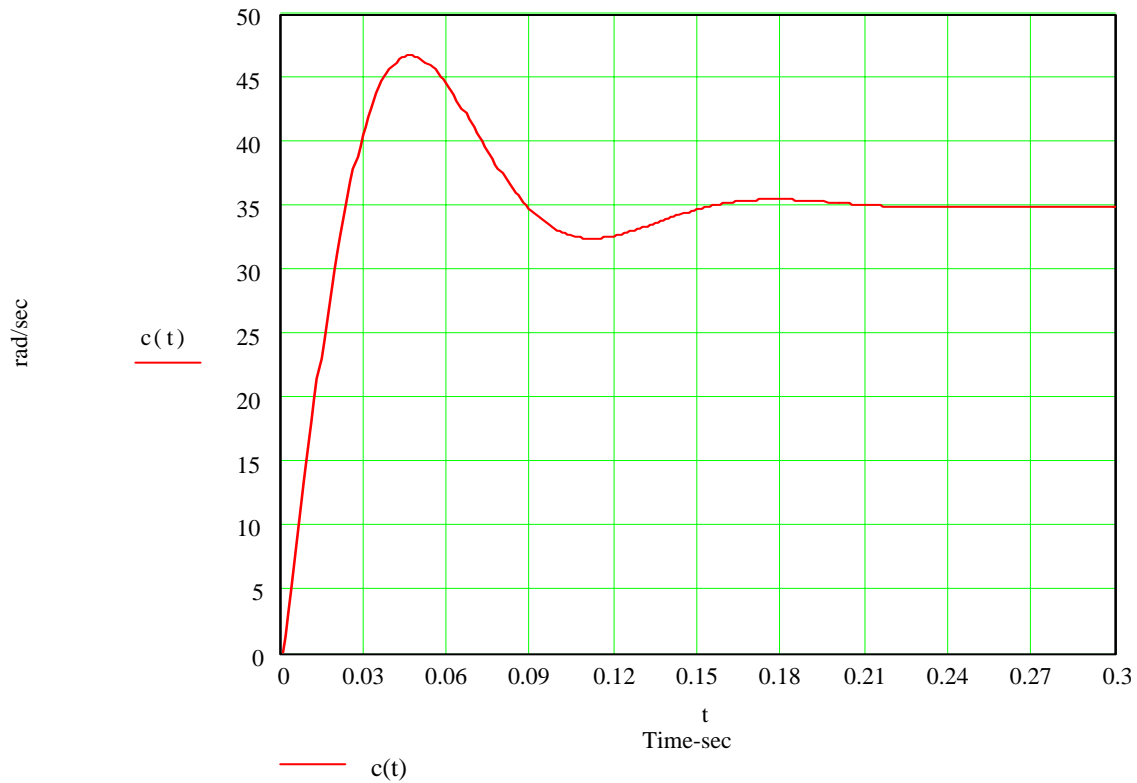


Fig 19 Vel. servo trans. resp.

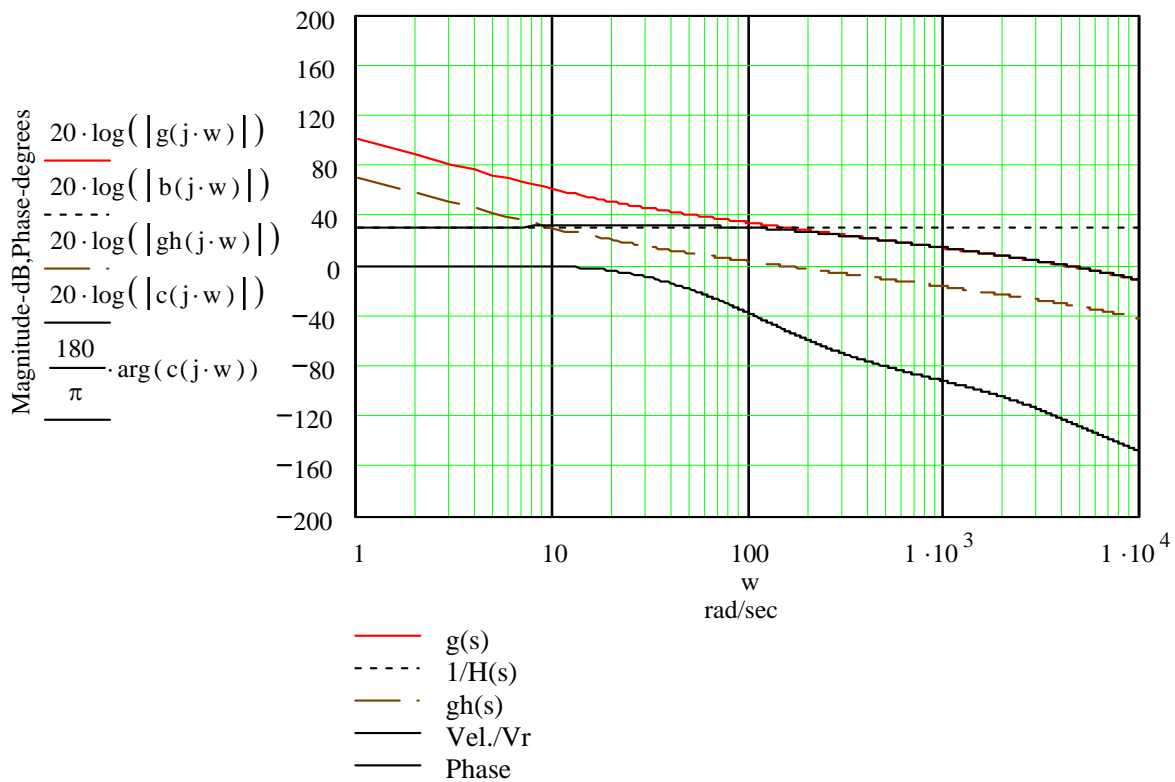


Fig. 20 Velocity servo response

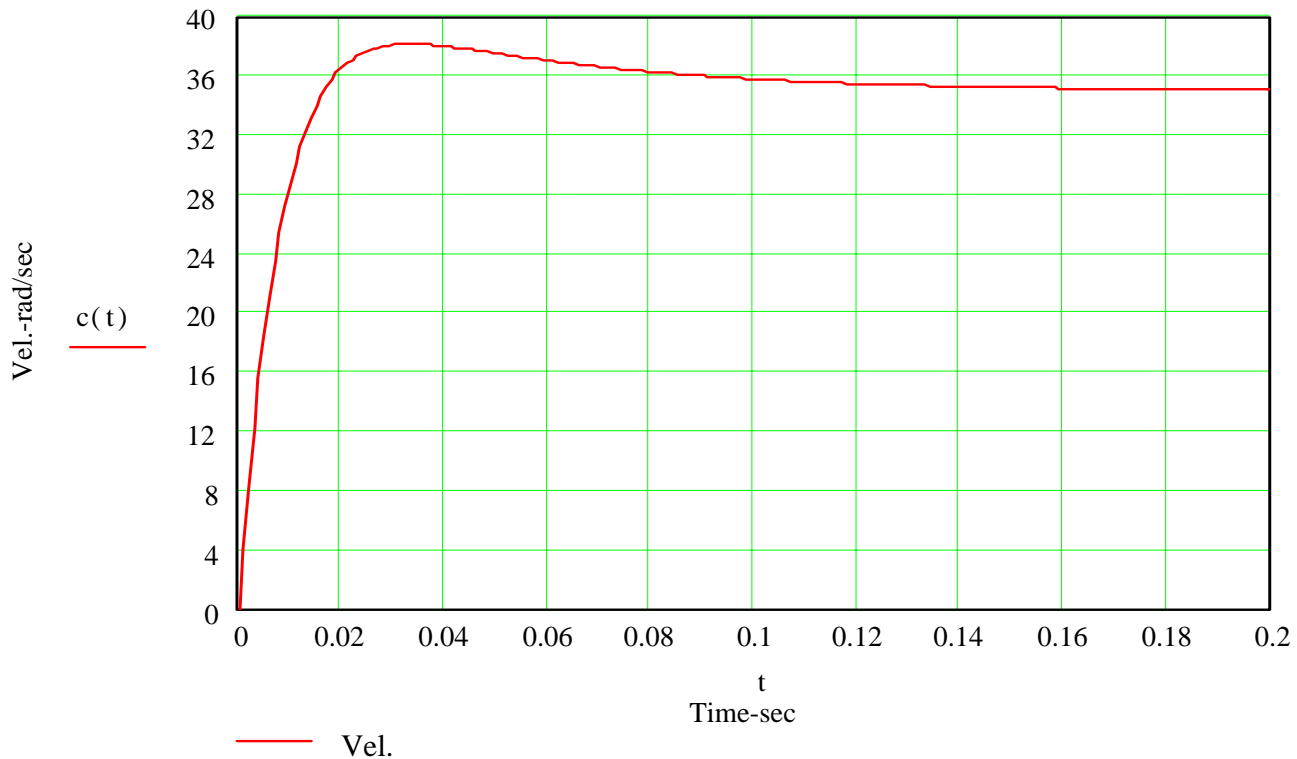


Fig. 21 Vel. servo trans. resp.

POSITION SERVO LOOP COMPENSATION

Having compensated the velocity servo, it remains to close the position servo around the velocity servo. Commercial industrial positioning servos do not normally use any form of integral compensation in the position loop. This is referred to as a “naked” position servo loop. However, for type 2 positioning drives, PI compensation would be used in the forward position loop. There are also some indexes of performance rules for the separation of inner servo loops by their respective bandwidths³. The first index of performance is known as the 3 to 1 rule for the separation of a machine resonance from the inner velocity servo. All industrial machines have some dynamic characteristics, which include a multiplicity of machine resonances. It is usually the lowest mechanical resonance that is considered; and the index of performance is that the inner velocity servo bandwidth should be 1/3 lower than the predominant machine structural resonance.

A second index of performance is that the position servo velocity constant (K_v) or position loop gain, should be 1/2 the velocity servo bandwidth³. These indexes of performance are guides for separating servo loop bandwidths to maintain some phase margin and overall system stability. Industrial machine servo drives usually require low position loop gains to minimize the possibility of exciting machine resonances. In general for large industrial machines the position loop gain (K_v) is set about 1 ipm/mil (16.66/sec). The example being studied in this discussion has a machine slide weight of 50,000 lbs., which can be considered a large machine that could have detrimental machine dynamics. There are numerous small machine applications where the position loop gain can be increased several orders of magnitude. The technique of using a low position loop gain is referred to as the “soft servo”. A low position loop gain can be detrimental to such things as servo drive stiffness and accuracy. The “soft servo” technique also requires a high-performance inner velocity servo loop. This inner velocity servo loop with its high-gain forward loop, overcomes the problem of low stiffness. For example, as the

machine servo drive encounters a load disturbance the velocity will instantaneously try to reduce, increasing the velocity servo error. However the high velocity servo forward loop gain will cause the machine axis to drive right through the load disturbance. This action is an inherent part of the drive stiffness³.

For this discussion it will be assumed that the industrial machine servo drive being considered has a structural mechanical spring/mass resonance inside the position loop. The machine as connected to the velocity servo drive is often referred to as the “servo plant”. The total machine/servo system can be simulated quite accurately to include the various force or torque feedback loops for the total system⁴. For expediency in this discussion, a predominant spring/mass resonance will be added to the output of the velocity servo drive. Thus the total servo system is shown in the block diagram of figure 22. Position feedback is measured at the machine slide to attain the best position accuracy.

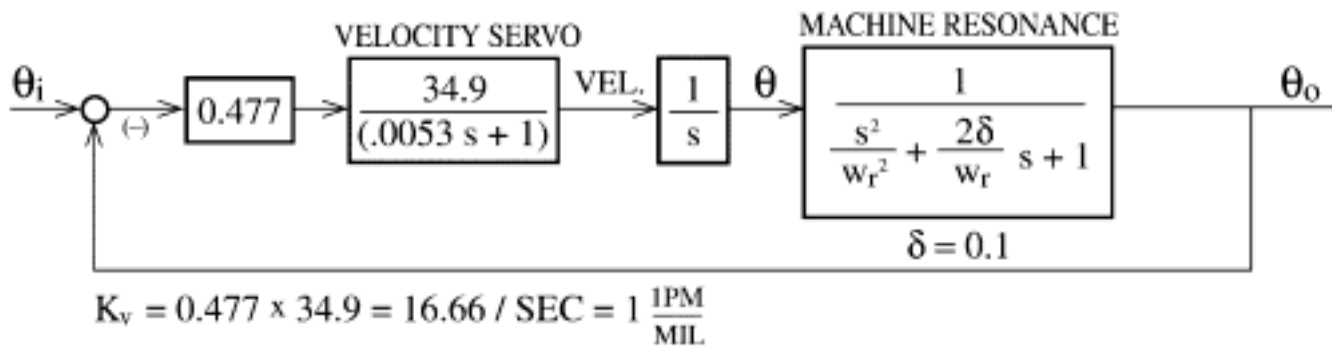


FIGURE 22 POSITION LOOP BLOCK DIAGRAM

As stated previously, the index of performance for the separation of the velocity servo bandwidth and the predominant machine resonance should be 3 to 1; where the velocity servo bandwidth is lower than the resonance. The machine resonance is shown in figure 22 as w_r . Since the velocity servo bandwidth of this example is 30 Hz (188 rad/sec) as in figure 20, the lowest machine resonance should be three times higher or 90 Hz (565 rad/sec). It is further assumed that this large machine slide has roller bearing ways with a coefficient of friction = 0.01 lbf/lb, and a damping factor ($\delta = 0.1$). Additionally, this industrial servo driven machine slide (50,000 lbs) will have a characteristic velocity constant (K_v) of 1 ipm/mil (16.66/sec). This large machine was used for this discussion as a worst case scenario since the large weight aggravates the reflected inertia and machine dynamics problems. Most industrial machines are not of this size. The closed loop frequency response with a mechanical resonance (w_r) of 90 Hz (565 rad/sec) is shown in figure 23. A unity step in position transient response is shown in figure 24. These are acceptable servo responses for the machine axis being analyzed.

In reality a machine axis weighing 25 tons will have structural resonances much lower than 90Hz. Machine axes of this magnitude in size will characteristically have structural resonances of about 10Hz to 20Hz. Using the same position servo block diagram of figure 22 with the same position loop gain of 1 ipm/mil (16.66/sec), and a machine resonance of 10 Hz; the servo frequency response is shown in figure 25 with the transient response shown in figure 26. The position servo frequency response shows a 8 dB resonant (62.8 rad/sec) peak over zero dB, which will certainly be unstable as observed in the transient response. Repeating the position servo analysis with a 20 Hz resonance in the machine structure, results in an oscillatory response as shown in figure 27 and 28.

One of the most significant problems with industrial machines is in the area of machine dynamics. Servomotors and their associated amplifiers have very long mean time before failure characteristics. It is

quite common to have an industrial velocity servo drive with 20 Hz to 30 Hz bandwidths mounted on a machine axis having structural dynamics (resonances) near or much lower than the internal velocity servo bandwidth. There must be some control concept to compensate for these situations. There are a number of control techniques that can be applied to compensate for machine structural resonances that are both low in frequency and inside the position servo loop. The first control technique is to lower the position loop gain (velocity constant). Depending on how low the machine resonance is, the position loop gain may have to be lowered to about 0.5 ipm/mil (8.33/sec.). This solution has been used in numerous industrial positioning servo drives. However, such a solution also degrades servo performance. For very large machines this may not be acceptable. The index of performance that the position loop gain (velocity constant) should be lower than the velocity servo bandwidth by a factor of two, will be compromised in these circumstances.

A very useful control technique to compensate for a machine resonance is the use of Wien bridge notch filters³. These notch filters are most effective when placed in cascade with the position forward servo loop, such as at the input to the velocity servo drive. These notch filters should have a tunable range from approximately 5 Hz to a couple of decades higher in frequency. The notch filters are effective to compensate for fixed machine structural resonances. If the resonance varies due to such things as load changes, the notch filter will not be effective. There are commercial control suppliers that incorporate digital versions of a notch filter in the control; with a future goal to sense a resonant frequency and tune the notch filter to compensate for it. This control technique can be described as an adaptive process.

Another technique that has been very successful with industrial machines having low machine resonances, is known as “frequency selective feedback^{5,6,7}”. This control technique is the subject of another discussion. In abbreviated form it requires that the position feedback be located at the servo motor eliminating the

mechanical resonances from the position servo loop, resulting in a stable servo drive but with significant position errors. These position errors are compensated for by measuring the machine slide position through a low pass filter; taking the position difference between the servo motor position and the machine slide position; and making a correction to the position loop; which is primarily closed at the servo motor.

Conclusions

Commercial industrial electric brushless DC servo drives use an inner current/torque loop to provide adequate servo stiffness. The servo loop bandwidth for the current loop is usually about 1000 Hz. In analysis this servo loop is often neglected because of its wide bandwidth. Including the current loop as in figure 2, results in a motor and current loop response (figure 7) that is an integration with the current loop response. A classical servo technique is to enclose the motor/current loop in a velocity servo loop. Since most commercial industrial brushless DC servo drives have position feedback from the motor armature for the purpose of current commutation; this signal is differentiated to produce a synthetic velocity loop. Additionally, commercial industrial servo drives use proportional plus integral (PI) servo compensation to stabilize the synthetic velocity loop.

The PI type of compensation has a corner frequency that must be a decade or more lower in frequency than the 45 degree phase shift frequency of the uncompensated open loop Bode plot. This requirement is needed to avoid excessive phase lag from the PI compensation where the open loop 45-degree phase shift frequency occurs. Commercial industrial electric servo drives have a very long mean time between failure, and

therefore are very reliable. The servo plant (the machine that the servo drive is connected to) has consistent problems with structural dynamics. When these mechanical resonances have low frequencies that occur within the servo bandwidths, unstable servo drives can result. The solution to such a problem can be a degradation in performance by lowering the position loop gain; using notch filters to tune out the resonance; or using a control technique referred to as “frequency selective feedback”^{5,6,7}.

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Figure 23 and 24

$$\omega_r = 90Hz \quad \delta = .1$$

$$g_{(s)} = \frac{16.66}{s(.0053s + 1)(.0000031s^2 + .000354s + 1)}$$

$$h_{(s)} = 1$$

Figure 25 and 26

$$\omega_r = 10Hz \quad \delta = .1$$

$$g_{(s)} = \frac{16.66}{s(.0053s + 1)(.000253s^2 + .003128s + 1)}$$

$$h_{(s)} = 1$$

Figure 27 and 28

$$\omega_r = 20Hz \quad \delta = .1$$

$$g_{(s)} = \frac{16.66}{s(.0053s + 1)(.000064s^2 + .0016s + 1)}$$

$$h_{(s)} = 1$$

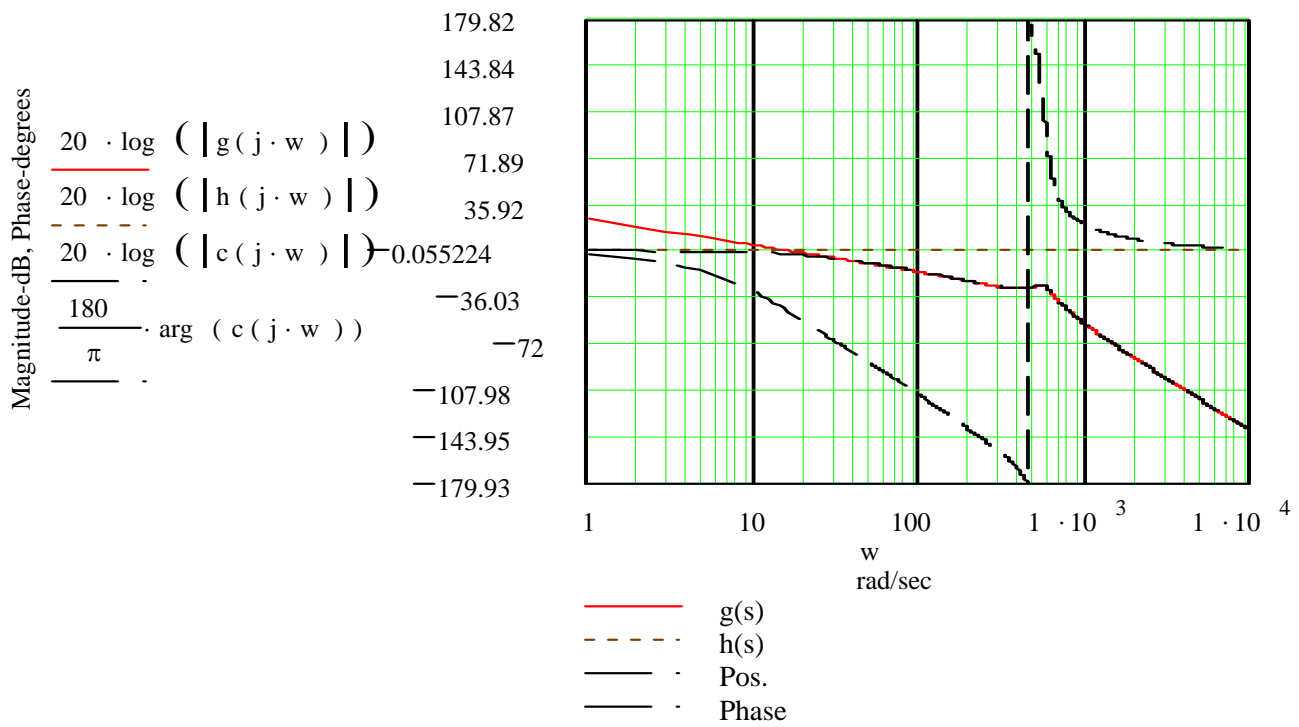


Fig 23 Position loop response- $W_r=90\text{Hz}$

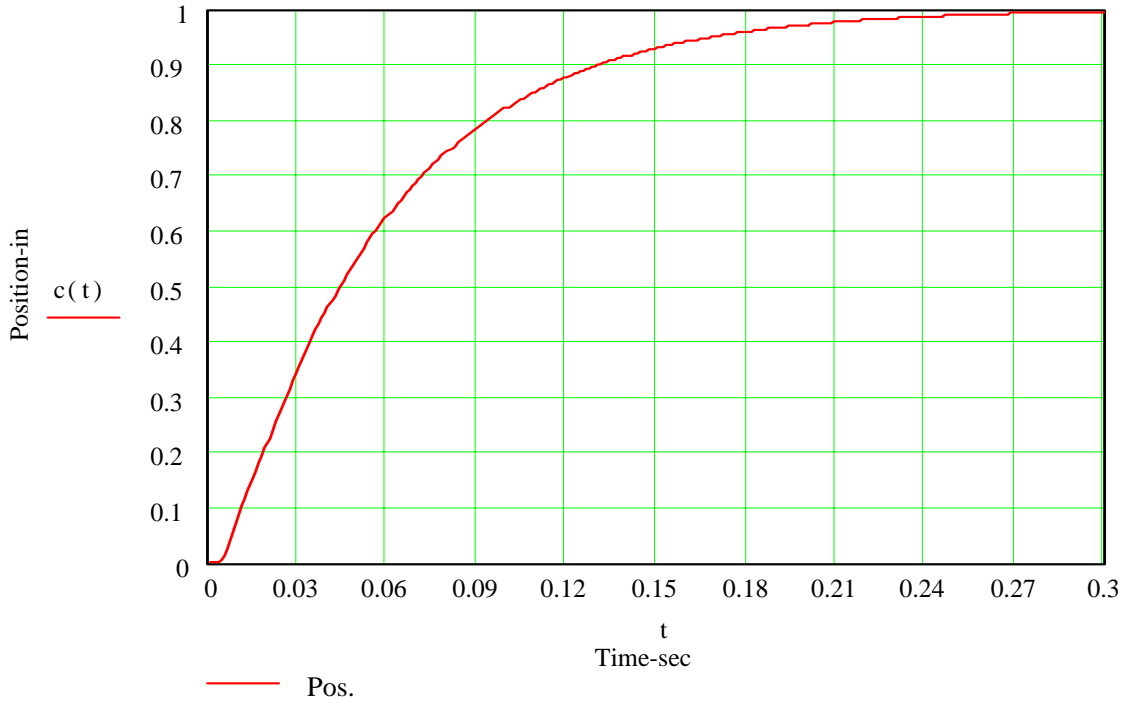


Fig 24 Pos. transient response

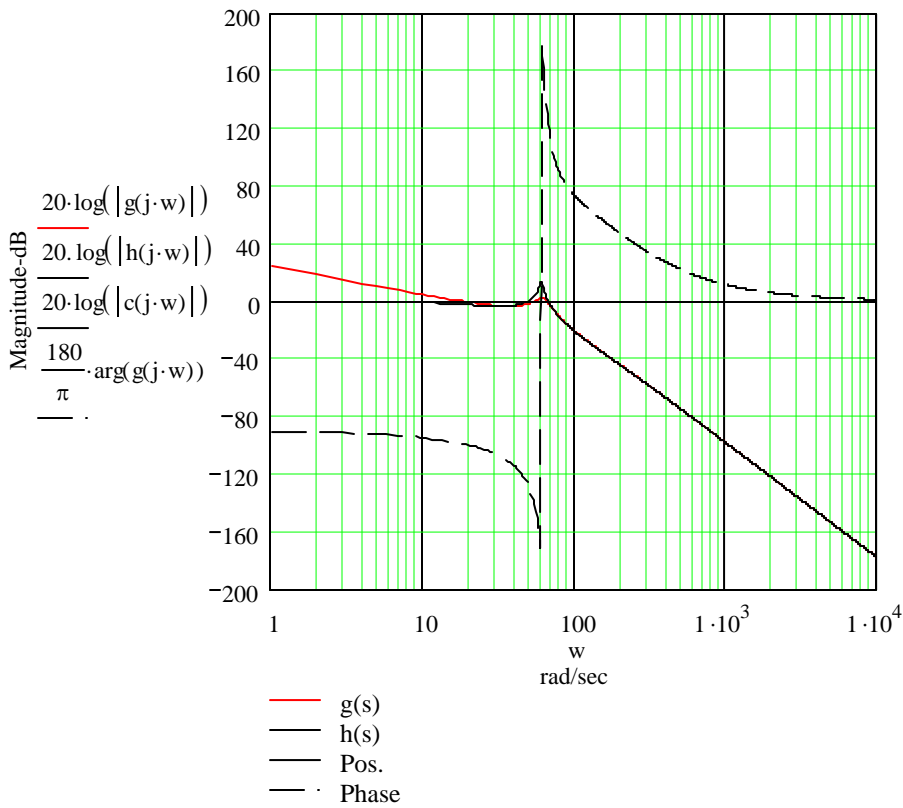


Fig 25 Pos. loop response- $W_r=10\text{Hz}$

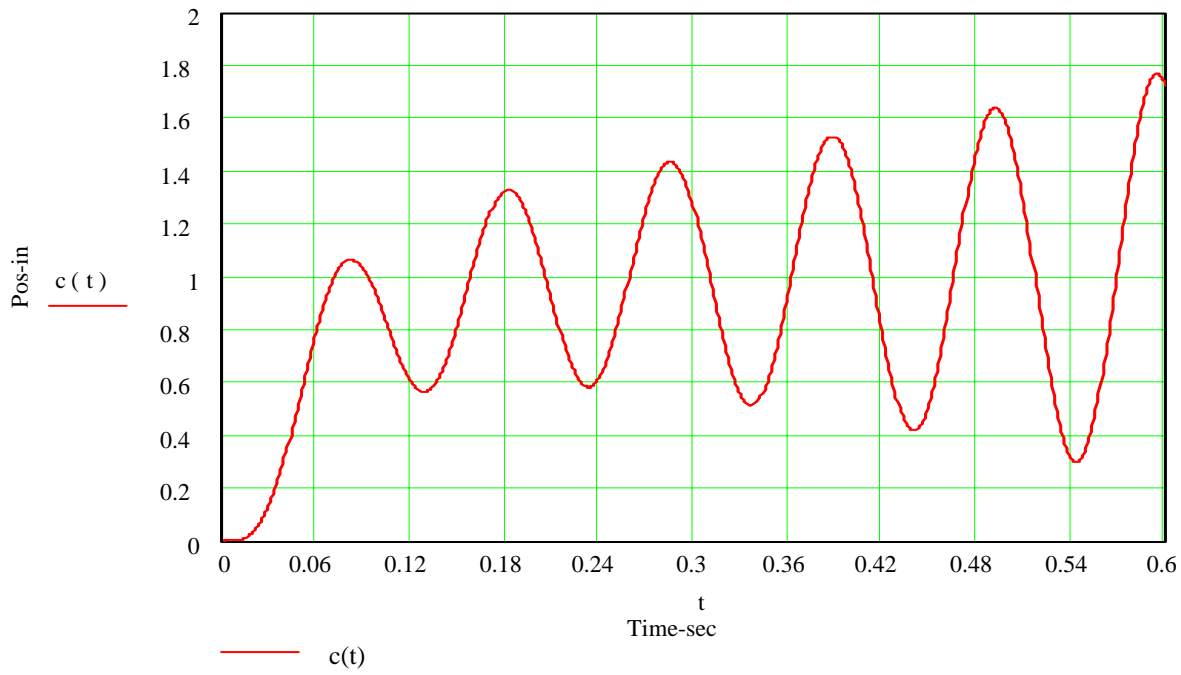


Fig 26 Pos. transient response

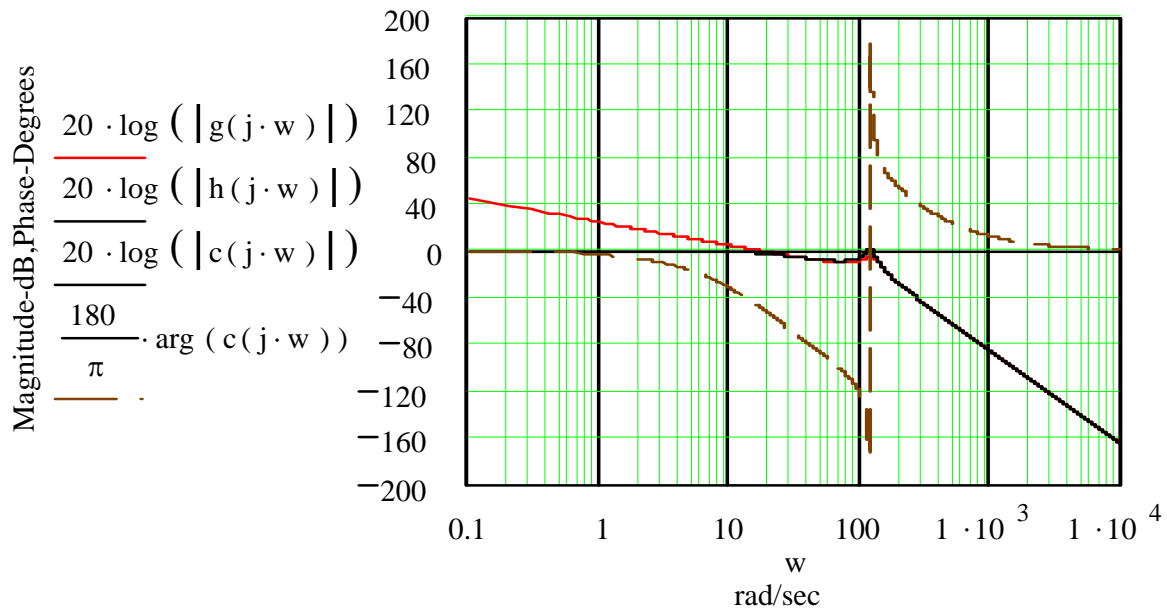


Fig. 27 Pos. loop response- $\omega_r=20\text{Hz}$

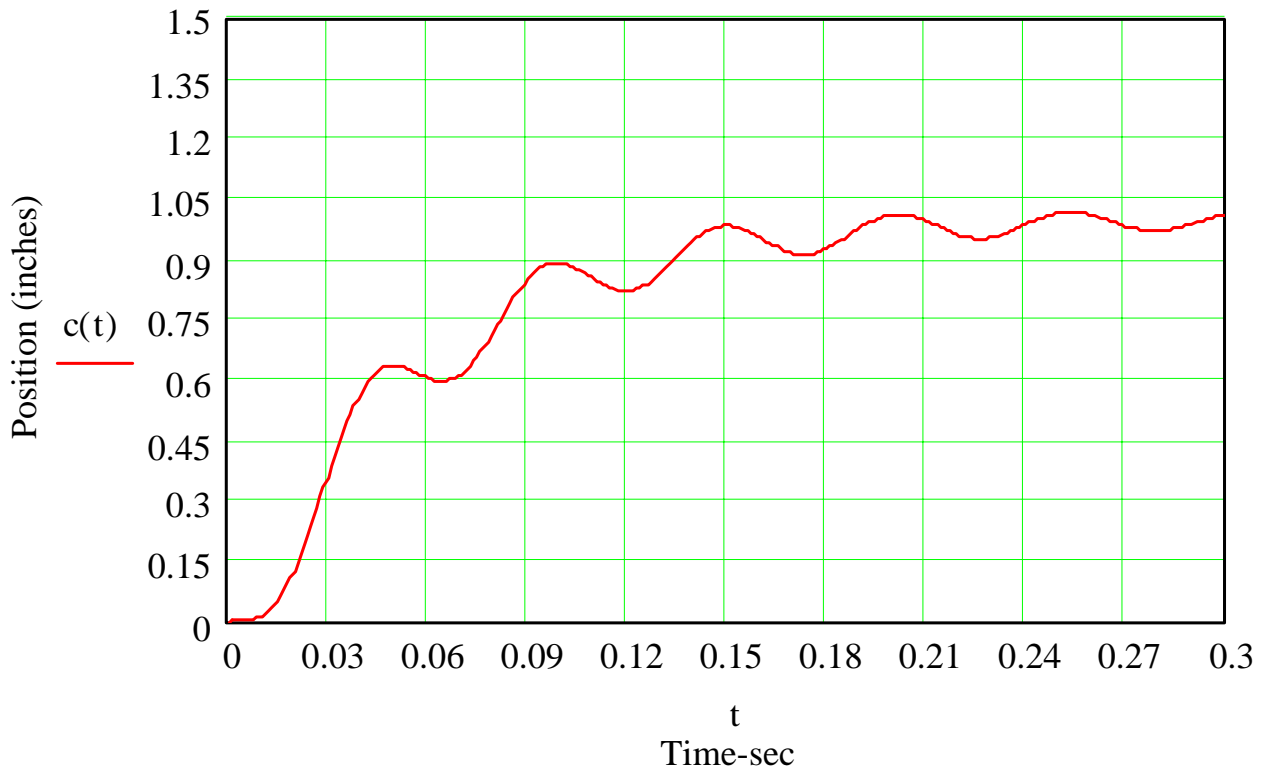


Fig 28 Transient resp.- $\omega_r=20\text{Hz}$