CACULATING P,I CONSTANTS Kp and Ki

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PI compensation is used with most industrial servos. D compensation is used when needed. An electric motor can be represented by a second order equation (1)

$$\frac{V_m}{e_i} = \frac{1/K_e}{s^2 + 2\delta\omega_n s + \omega_n^{\ 2}} \tag{1}$$

Where: Ke= [volts/rad/sec]

$\omega_n = bandwidth\ of\ \ motor\ drive = [rad/\sec]$

An index of performance can be written to equate with the *characteristic equation* of eq.1 such that:

$$\frac{((K_p/J)s + (K_i/J))}{[s^2 + (K_p/J)s + K_i/J]} \tag{2}$$

Equating coefficients of eq. 1 and eq. 2 :

$$2\delta\omega_n = \frac{K_p}{J} \tag{3}$$

and

$$\omega_n^{\ 2} = \frac{K_i}{J} \tag{4}$$

Where:

J= Jmotor+Jload reflected to motor= $[lb - in - \sec^2]$

$\omega_n = motor\ drive\ bandwidth = rad/\sec$

$\delta = damping\ \ factor$

It can be assumed that the drive will be well damped, thus $\delta = 1$

The bandwidth should be chosen as a reasonable value of approximately 20 to 30 Hz (125 to 188 rad/sec) for industrial machines.

Therefore, substituting these approximations into eq. 3 and eq. 4 yields

$2\omega_n = \frac{K_p}{J}$ and $\omega_n^{\ 2} = \frac{K_i}{J}$

$K_p = 2\omega_n J$ [a/rpm] and $K_I = \omega_N^{\ 2} j$ [a/sec/rpm]

EXAMPLE:
An industrial machine servo has the following characteristics-

Jtotal at motor=0.311 $lb-in-\sec^2$

The drive response is desired to respond with a damping factor as close to critical damping as possible making $\delta=1$.

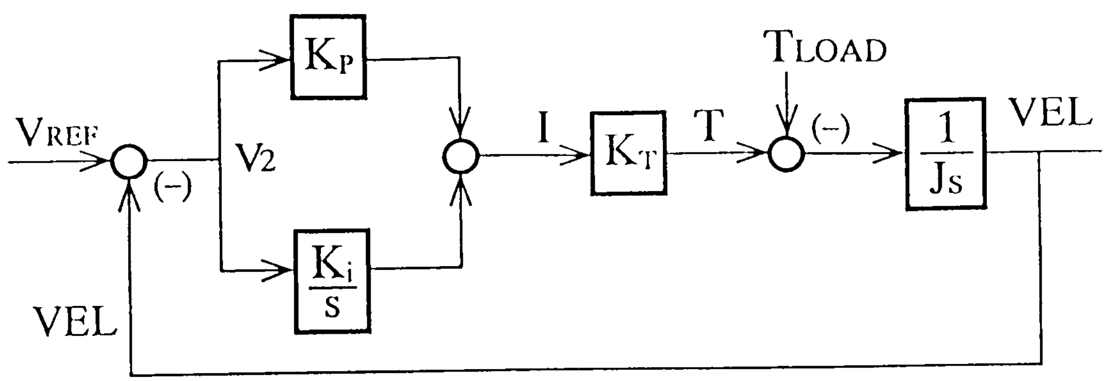
The normal servo bandwidth of this industrial servo will be about 30 Hz (188 rad.sec).

Therefore:

$$K_p = 2x188x0.311 = 118 \text{ [a/rpm]}$$

$$K_i = 188^2 \ x0.311 = 11000 \text{ [a/sec/rpm]}$$

The PI compensation can be represented as-

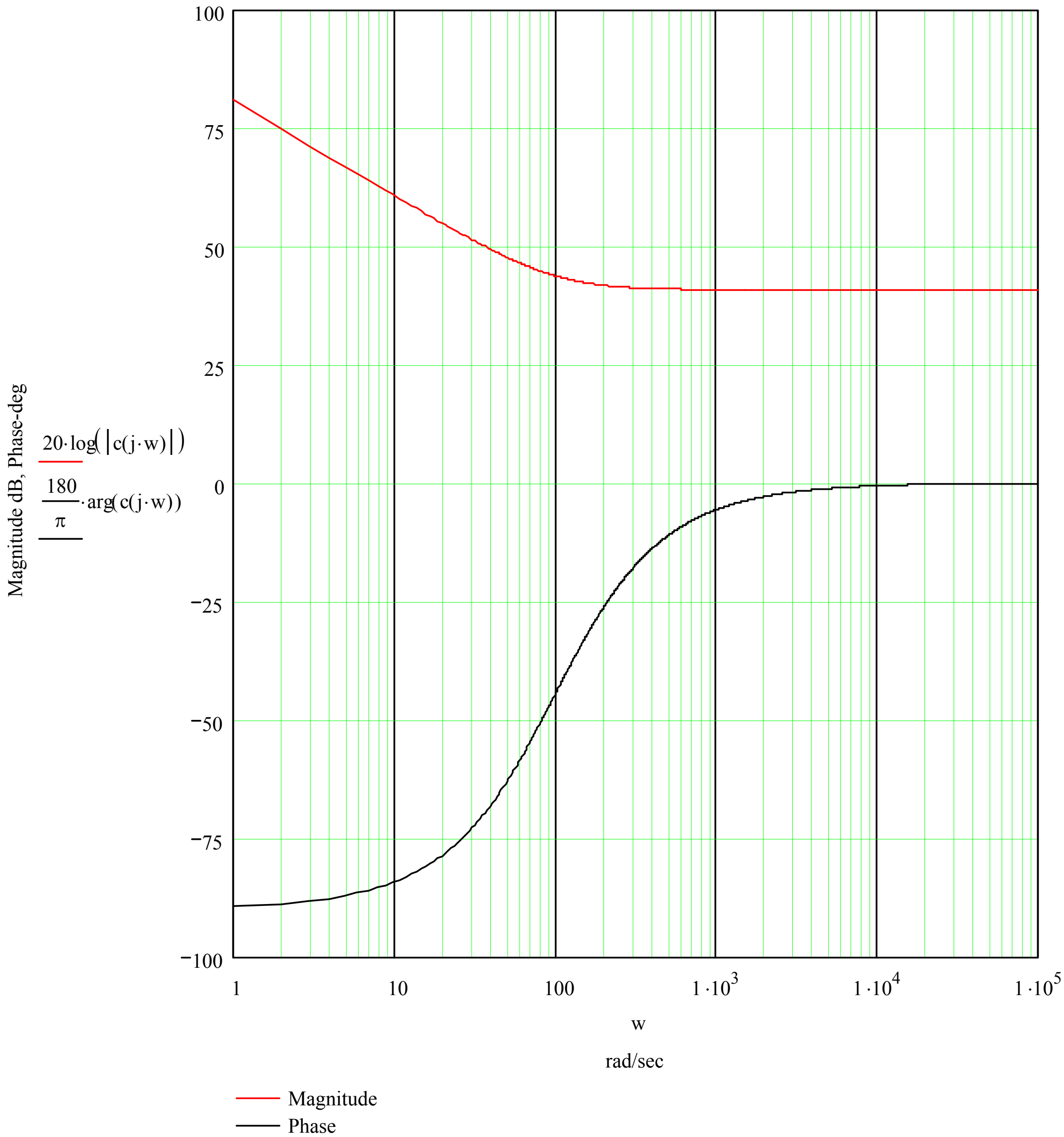


$$\frac{I}{V_2} = \left[K_p + \frac{K_i}{s}\right] = \frac{K_p s + K_i}{s} = \frac{K_i\left[\frac{K_p}{K_i}s+1\right]}{s} = \frac{K_2[t_2 s+1]}{s}$$

$$t_2 = \frac{K_p}{K_i} \qquad w_2 = \frac{K_i}{K_p} \quad \text{(Corner frequency)}$$

$$\omega_2 = \frac{K_i}{K_p} = \frac{11000}{117} = 94 \ rad/\sec$$

The frequency response for PI compensation is as follows:



PI Compensation