ELECTRIC VELOCITY SERVO REGULATION

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The performance of an electrical velocity servo is a measure of how well the servo drive will maintain its commanded velocity under varying load disturbances. The ability to maintain a velocity under load changes is expressed as the regulation of the servo drive usually expressed as percent regulation. A typical electric servo drive with a current loop for torque regulation is shown in the block diagram of figure 1. Since the two servo loops are interacting, block diagram algebra is used to rearrange the block diagram into two independent servo loops as shown in figure 2.

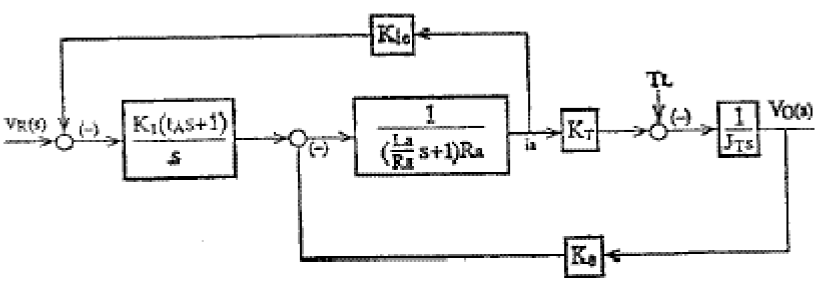

Fig. 1 Electric servo-drive block diagram.

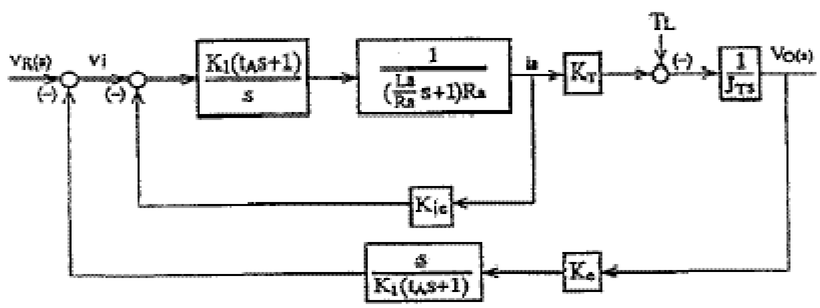

Fig. 2 Electric servo-drive block diagram.

The rearranged block diagram of the motor and current loop will then be included into a velocity feedback servo loop shown in figure 3. The motor and current servo loop has PI compensation. The external velocity servo loop is shown with a lag/lead compensation but could just as well also be PI compensation. Most commercial electric servos also have an added position servo loop, which can increase the velocity regulation to a very large extent. This discussion is limited to an electrical velocity servo, which can be a dc servo drive or a brushless dc (BLDC) servo drive. Speed regulation with an added position servo loop is the subject of another discussion.

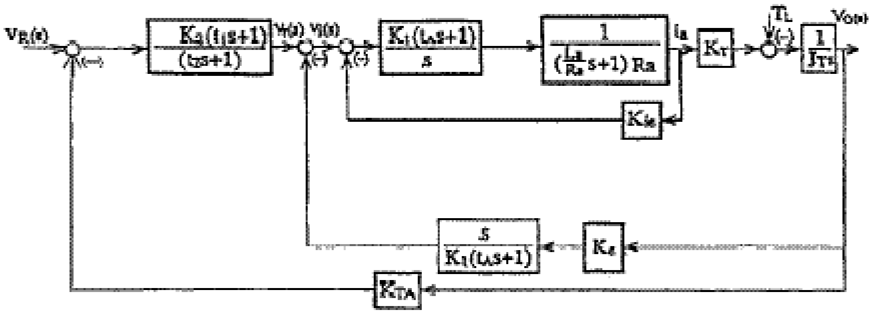

Fig. 3 Electric servo-drive block diagram.

To discuss the regulation of a velocity servo drive, the block diagram of figure 3 is rearranged in figure 4 to show motor velocity (V_o) as a function of torque load (T_l) changes. The inner current servo loop of figure 3 is expressed as eq.1. For the steady state condition the current servo loop is given as eq. 2.

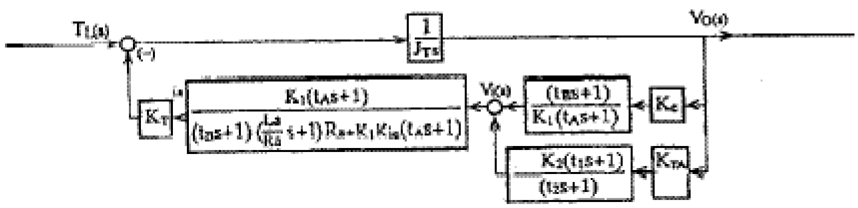

Fig. 4 Electric servo-drive block diagram.

$$\frac{i}{v_i} = \frac{K_1(t_a s+1)}{(t_b s+1)(\frac{l}{r}s+1)R_a + K_1(t_a s+1)K_{ie3}} \qquad \text{eq. (1)}$$

$$\frac{i}{v_i}\,\frac{}{s \to 0} = \frac{K_1}{R_a + K_1 K_{ie}} \qquad \text{eq (2)}$$

The external velocity servo loop is given in eq (3) with the steady state condition as eq (4).

$$\frac{V_o}{v_r} = \frac{K_1(t_a s+1)K_t}{[t_b s+1)(\frac{l}{r}s+1)R_a + K_1(t_a s+1)K_{ie}]J_t s + K_1(t_a s+1)K_t \dfrac{(t_b s+1)K_e}{K_1(t_a s+1)}} \qquad \text{eq (3)}$$

$$\frac{V_o}{v_r}\underset{s \to 0}{\overline{\qquad}} = \frac{K_1 K_t}{K_1 K_t \frac{K_e}{K_1}} = \frac{K_1}{K_e} \qquad \text{eq (4)}$$

A readily available parameter of commercial servo drives is shown as the open loop gain (K_{vo}) of the velocity servo. Equation (5) is the open velocity servo loop gain (K_{vo}) with the steady state solution in eq (6).

$$K_{vo} = \frac{K_{TA} K_2 (t_1 s + 1) K_1 (t_a s + 1) K_t}{(t_2 s + 1)[(t_b s + 1)(\frac{l}{R})s + 1)R_a + K_1 K_{ie}(t_a s + 1)]J_t s + K_1 (t_a s + 1)\frac{K_t (t_b s + 1) K_e}{(t_a s + 1) K_1}} \qquad \text{eq (5)}$$

$$K_{vo} = \frac{K_{TA} K_2}{\frac{K_e}{K_1}} = \frac{K_{TA} K_1 K_2}{K_e} \qquad \text{eq (6)}$$

The drive regulation can be computed from figure 4 with eq (7) and eq (8) for the steady state solution.

$$\frac{V_o}{T} = \frac{1}{J_t s + \frac{K_t K_1 (t_a s + 1)}{[(t_b s + 1)(\frac{L}{R} s + 1)R_a + K_1 K_{ie}(t_a s + 1)]} + [\frac{K_e (t_b s + 1)(t_2 s + 1) + K_2 K_{TA} K_1 (t_1 s + 1)(t_a s + 1)}{(t_a s + 1) K_1) t_2 s + 1)}]}$$

eq (7)

$$\frac{V_o}{T}\underset{s \to 0}{\overline{\qquad}} = \frac{1}{\frac{K_t K_1}{R_a + K_1 K_{ie}} x \frac{K_e + K_2 K_{TA} K_1}{K_1}} \qquad \text{eq (8)}$$

Equation 8 is rearranged as given in eq 9. The open loop servo gain of eq (6) is rearranged in eq(10). Substituting eq (10) into eq (9) results in eq (11), which is simplified in eq (12).

$$\frac{V_o}{T}\underset{s \to 0}{\overline{\qquad}} = \frac{R_a + K_1 K_{ie}}{K_t [K_e + K_2 K_{TA} K_1]} = \frac{R_a + K_1 K_{ie}}{K_e K_t + K_1 K_2 K_t K_{TA}} \qquad \text{eq (9)}$$

$$K_{TA} K_1 K_2 = K_{vo} K_e \qquad \text{eq (10)}$$

$$\frac{V_o}{T} = \frac{R_a + K_1 K_{ie}}{K_e K_t + K_{vo} K_e K_t} \qquad \text{eq (11)}$$

$$\frac{V_o}{T} = \frac{R_a + K_I K_{ie}}{K_e K_I (1 + K_{vo})}$$ (Speed regulation equation) eq (12)

Dimensional Analysis

It is important to know the units of the parameters in eq. (12) for both dc and BLDC electric servos. The unit dimensions are shown as follows:

	DC DRIVES	BLDC DRIVES
R_a (Armature resistance)	$R_a[ohms]$	$\frac{R_{a(l-l)}}{2}[ohms]$
K_e (Voltage constant)	$[\frac{volts-\sec}{rad}]$	$\frac{K_{e(l-l)}}{\sqrt{3}}[\frac{volts-sec}{rad}]$
K_t)Torque constant)	$\left[\frac{lb-in}{rad}\right]$	$\left[\frac{lb-in}{rad}\right]$
K_{vo} (Open loop gain)	$\left[\frac{volt}{volt}\right]$	$\left[\frac{volt}{volt}\right]$

Dimensional analysis of eq 12

$$\frac{V_o}{T} = \frac{ohms + \frac{v}{v} x \frac{v}{a}}{\frac{v}{r/s} x \frac{lb-in}{a} x \frac{v}{v}} = [\frac{rad/sec}{lb-in}]$$

EXAMPLE

Motor – Kollmorgen M607B

$$\frac{R_{a(l-l)}}{2} = \frac{0.189[ohm]}{2} = 0.094[ohm]$$

$$\frac{K_{e(l-l)}}{\sqrt{3}} = \frac{0.646}{\sqrt{3}} = 0.3729\left[\frac{volt-sec}{rad}\right]$$

Rated Torque= 396[lb-in]

$$K_t = 9.9\left[\frac{lb-in}{A}\right]$$

Rated Speed= 3000[rpm]

K_{ie}= Current loop feedback constant = $\frac{3v}{40A} = 0.075\left[\frac{v}{A}\right]$

$$K_1 = 20\left[\frac{v}{v}\right]$$

$$K_{TA} = 0.0286\left[\frac{volt-sec}{rad}\right]$$

$$K_2 = 651\left[\frac{volt}{volt}\right]$$

$$K_{vo}\ (velocity\ open\ loop\ gain) = \frac{K_{TA}\ x\ K_1\ x\ K_2}{K_e} = \frac{0.0286\ x\ 20\ x\ 651}{0.3729} = 1000\left[\frac{volt}{volt}\right]$$

REGULATION

$$\frac{V_o}{T} = \frac{R_a + K_1 K_{ie}}{K_e K_t (1 + K_{vo})} = \frac{0.094 + 20\ x\ 0.075}{0.3729\ x\ 9.9\ x\ (1+1000)} = \frac{1.594}{3691} = 0.00043\left[\frac{volt}{lb-in}\right]$$

Speed drop at rated torque and rated speed-

$$Speed\ drop = 0.00043\left[\frac{rad/sec}{lb-in}\right] x\ 396\ [lb-in] = 0.17\left[\frac{rad}{sec}\right]$$

$$0.17\left[\frac{rad}{\sec}\right]x\left[\frac{rev}{2\pi\ rad}\right]x\left[\frac{60\sec}{\min}\right]=1.626\ [rpm]$$

REGULATION$=\dfrac{\textit{Change in speed}}{\textit{Rated speed}}=\dfrac{1.626\ [rpm]}{3000\ [rpm]}=0.000542$

Velocity servo **% REGULATION**= 0.000542 x 100= 0.0542 %

If a position loop is added to the velocity servo drive the block diagram is shown in figure (5) . The position loop velocity constant (K_v) is the position open loop gain.

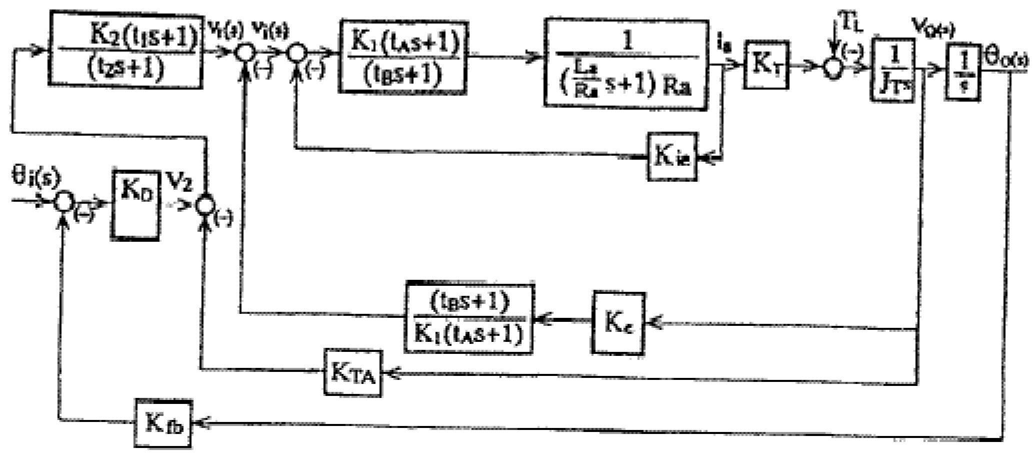

Fig. 5 Electric servo-drive block diagram.

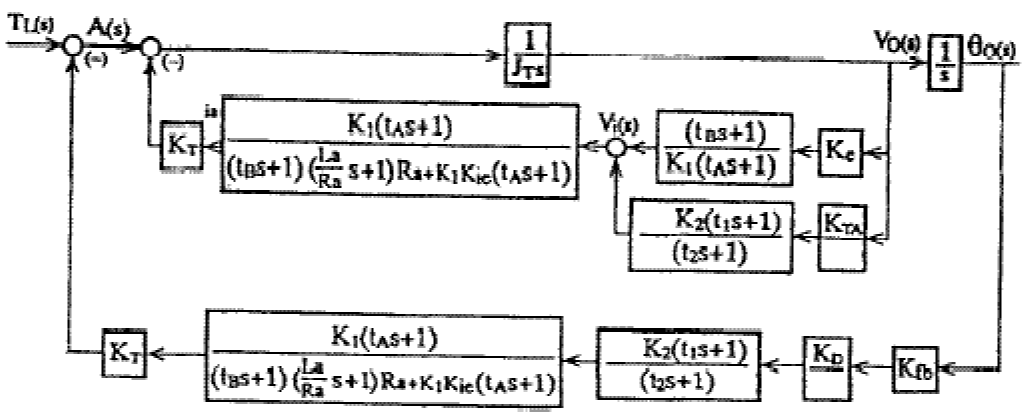

Fig. 6 Electric servo-drive block diagram.

$$K_v = K_D K_{fb} \; x \; \frac{V_o}{v_2} = K_D K_{fb} \; x \; \frac{K_1 K_2}{(K_e + K_1 K_2 K_t K_{TA})} \qquad \text{eq (13)}$$

Reducing eq (13) yields

$$K_v = \frac{K_1 K_2 K_D K_{fb}}{(K_e + K_1 K_2 K_{TA})} \qquad \text{eq (14)}$$

Rearranging eq (6) yields

$$K_e K_{vo} = K_1 K_2 K_{TA} \qquad \text{eq (15)}$$

Substituting eq (15) into eq (14) yields

$$K_v = \frac{K_1 K_2 K_D K_{fb}}{(K_e + K_e K_{vo})} = \frac{K_1 K_2 K_{fb}}{K_e (1 + K_{vo})} \qquad \text{eq (16)}$$

Rearranging yields

$$K_v (1 + K_{vo}) = \frac{K_1 K_2 K_D K_{fb}}{K_e} \qquad \text{eq (17)}$$

From figure 6, the position (θ) vs Torque (T) is expressed as-

$$\frac{\theta}{T} = \frac{R_a + K_1 K_{ie}}{K_t K_1 K_D K_{fb} K_2} \quad \text{(steady state compliance)} \qquad \text{eq (18)}$$

$$\frac{T}{\theta} = \frac{K_1 K_2 K_D K_t K_{fb}}{R_a + K_1 K_{ie}} \quad \text{(steady state stiffness)} \qquad \text{eq (19)}$$

Substituting eq (17) into eq (19) yields

$$\frac{T}{\theta} = \frac{K_v (1 + K_{vo}) K_e K_t}{R_a + K_1 K_{ie}} \qquad \text{eq (20)}$$

$$\frac{T}{\theta} = \frac{K_v (1 + K_{vo}) K_e K_t}{R_a (1 + \frac{K_1 K_{ie}}{R_a})} \qquad \text{eq (21)}$$

Figure 5 is the position servo block diagram. The input command and the output θ_o are in radians. However if the input command is a given position over a period of time; that is a velocity, and the output position follows the command with a lag and this lag is defined as the "following error" in a type 1 position servo. Therefore, if the input command and the output is differentiated (θs), the drive will be in a velocity mode expressed as-

$$\frac{\theta_o}{T} = \frac{V_o}{sT} = \frac{R_a + K_1 K_{ie}}{K_v(1+K_{vo})K_e K_t} \qquad [\frac{rad/sec}{lb-in}] \qquad \text{eq (22)}$$

Dimensional check of eq (22)

$$\frac{ohms + \frac{v}{v}\frac{v}{A}}{sec[\frac{1}{sec}\frac{v}{v}\frac{v}{\frac{rad}{sec}}\frac{lb-in}{A}]} = \left[\frac{rad/sec}{lb-in}\right]$$

EXAMPLE

Using the same motor as in the previous example with the same torque and velocity inner servo loop, the new variable is the position open loop gain (velocity constant) K_v. Thus the variables are repeated as-

R_a=0.094 [ohms]
K_e=0.3729 [volt-sec/rad]
Rated torque= 396 [lb-in]
K_t=9.9 [lb-in/amp]
Rated speed= 3000[rpm]
K_{ie}=0.075[volt/amp]
K_i= 20 [v/v]
K_{TA}=0.0286[volt-sec/rad]
K_2=651[volt/volt]
K_{vo}=1000[volt/volt]

The position loop gain will be K_v=1 [ipm/mill]= 16.66 [rad/sec]
The regulation of equatrion (22) can be calculated as-

$$\frac{V_o}{T} = \frac{R_a + K_1 K_{ie}}{K_v(1+K_{vo})K_e K_t} = \frac{0.094 + 20\,x\,0.075}{16.66(1+1000)0.3729\,x\,9.9} = 0.00002589$$

Speed drop at rated torque and rated speed-

$$Speed\ drop = 0.00002589\left[\frac{rad}{sec}\right] x\,396\left[lb-in\right] = 0.0102\left[\frac{rad}{sec}\right]$$

$$0.0102\left[\frac{rad}{sec}\right] x \left[\frac{rev}{2\pi\, rad}\right] x \left[\frac{60\,sec}{min}\right] = 0.0974\left[rpm\right]$$

$$\textbf{REGULATION} = \frac{change\,in\,speed}{rated\ speed} = \frac{0.0979[rpm]}{3000\|rpm\|} = 0.0000326$$

Position servo in velocity mode **% REGULATION**=0.0000326 x 100= 0.00326%

Reference: G. Younkin, INDUSTRIAL SERVO CONTROL SYSTEMS-Fundamentals and Applications, 2nd Edition, Chapter 12, 2002, Marcel Dekker, Inc, N.Y.,N.Y.